Global Review of Accounting and Finance
Vol. 11. No. 1. September 2020 Issue. Pp. 15– 27

The Cross-Section of Expected Returns: A Case of over Aggregation

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Beta, as a measure of risk, is used by many investors and academicians for forecasting the expected earnings. Fama and French (1992) first used data of New York Stock Exchange (NYSE), American Stock Exchange (AMEX), and National Automated Security Dealers Automated Quotes (NASDAQ) from July 1963 to December 1990 in the univariate regression model. They formed ten size-based portfolios and then divided each of these ten portfolios into ten sub-portfolios based on beta. They found that the size-beta portfolio returns show a negative relationship with portfolio betas. Hence, it was concluded that beta does not describe the average stock market returns. In the appendix of their paper they extended the time period to fifty years covering from 1941 to 1990 and arrived on the same results. Hence, it was concluded that beta cannot be salvaged. Many other studies arrived on similar conclusions using data from different countries.

This paper is using the data of NYSE, AMEX, and NASDAQ following the methodology of Fama and French (1992), for a period of 28½ years from July 1964 to December 1992, arrived on the same results as those of Fama and French (1992) that beta does not describe the average stock market returns. After this, many other studied arrived on the similar conclusions in different markets from Latin America to Indonesia. In this paper we made slight adjustment to the data set by dividing the total time period into two parts based on market condition (bear and bull). Using the same model, we will show that beta is still alive, and it is more powerful than previously thought. This result shows that the conclusion of Fama and French (1992) are not the problem of beta, instead it is a problem of over aggregations.

JEL: E44, F65, G11, G12, and G32

Key Terms: Beta, CAPM, Risk Premia, Bull Markets, Bear Markets.

1. Introduction

Jhamb and Menani (2019) tested the Capital Asset Pricing Model (CAPM) using the data of 170 companies listed on the Bombay Stock Exchange and conclude that CAPM is not suitable in the Indian context for the period of 2000 to 2016. Saleh (2020) tested CAPM in the Indonesian markets covering a period of four years from 2012 to 2016 and found that the market return has positive relationship, but it is only partially significant. Asgharian and Hansson (2002) tested CAPM in Swedish market and conclude that beta is never significantly different from zero. Hodoshima, Garza-Gomez and Kunimura

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(2000) using cross-sectional regression method on the data from Japan found a flat relationship between return and beta. Sandoval and Saens (2004) used conditional cross-sectional CAPM) on the data from four Latin American countries namely, Argentinean, Brazilian, Chilean and Mexican stock markets and conclude that the relationship between return and beta is dominant. Phong and Hoang (2012) found that three factor model presented by Fama and French (1993) is better than CAPM in depicting the relationship between returns and beta on the data from Stock Exchange of Vietnam.

The Capital Asset Pricing Model (CAPM) of Sharp (1964), Mossin (1966), Lintner (1965), and Black (1972) is widely used by academicians and practitioners for predicting average returns and risk. This model is mathematically stated as follows:

$$E(R_i) = \beta_i (R_m - R_f) + R_f$$
$$\beta_i = [\, Cov(R_m , R_i) / Var(R_m) \,] ,$$

where β_i is the non-diversifiable (rewarded/compensated) risk, $E(R_i)$ is the expected return of asset i, R_m is the return on market portfolio, R_f is risk free rate, and $(R_m - R_f)$ is the risk premia.

Many financial economists using CAPM have found that risk-adjusted stock returns are higher for smaller firms than for larger firms and termed it as size effect. For example, a size effect is found by Banz (1981), Reinganum (1981), and Keim (1983) for NYSE and AMEX; and by Lamoureux and Sanger (1989) in the Over the Counter (OTC) market. Size effects for foreign markets also have been found. For example, a size effect for the UK has been documented by Dimson and Marsh (1986) and for Australian stock market by Brown, Keim, Kledon, and Marsh (1983) documented the size effect. Fama and French (1992) analyzed stock market data for fifty years and concluded the relations between equity beta and average return is flat, even when beta is the only explanatory variable. This paper first attempts to see the relationship between beta and average return and arrived on the conclusion like Fama and French (1992). Then this study incorporated the market conditions (bull and Bear) in the model as suggested by Bhardwaj and Brooks (1993) and arrived on completely opposite results from Fama and French (1992). Main result of this paper is to show that the conclusion arrived by Fama and French (1992) is the result of aggregation over a long period of time. Beta, if used properly, is a predictor of average returns and is a much stronger predictor than previously documented. This study, now, needs to be replicated using current data from stock markets of different countries for further validation of our results.

This paper is organized in four parts, Introduction in Section 1 is followed by the Literature Review in Section 2. Data and Methodology are presented in Section 3 . Section 4 focuses on conclusion and Implication

2. Literature Review

Banz (1981) found that smaller firms have higher risk-adjusted returns than larger firms, by analyzing a period of forty year of data from NYSE. Roll (1981) used a single factor

CAPM and multi-factor, Arbitrage Pricing Theory (APT), to investigate the size effect and conclude that the errors in prediction are due to autocorrelation caused by thin trading, which is more for smaller firms than larger firms. Reinganum (1981) finds that a firm size effect still exists even after controlling for earning/price differences across firms. Basu (1983), on the other hand, argues that the earning/price effect subsumes the size effect. Stoll and Whaley (1983) find that transaction costs partially account for the abnormal returns of small firms. Keim (1983) finds that half of the annual firm size effect occurs in January. Chan, Chen and Hsieh (1985) investigated the firms size effect for stocks traded on NYSE from 1953 to 1977 using a multi-factor asset model (which is like the Arbitrage Pricing Model developed by Ross, 1976). They find that deviations of actual returns from required returns are higher for small firms than for larger firms, but the difference is insignificant. Chan and Chen (1988) attribute the size effect to a measurement error in beta. They argue that size is a proxy that captures the measurement error in beta. Stoll and Whaley (1983) find that transaction costs partially account for the abnormal returns of small firms. Barry and Brown (1984) find that the firm size effect is not explained by the informational differential between small and large firms. Kross (1985) decomposed size into two components, share price and share outstanding and finds that price is the dominating variable in explaining the cross-sectional variance in excess returns from small firms. Jagadeesh (1992) shows that because of high correlation between beta and firm size, the differences in the performance of size based portfolios cannot be attributed unambiguously either to firm size or beta.

Reinganum (1983) uses a tax-loss selling argument to explain this seasonal effect. He proposes that the shares of firms losing value during the year are sold in December for a tax write-off. These stocks, then, often are repurchased in January of the next year, placing upward pressure on stock prices. Brown, Keim, Kleidon, and Marsh (1983) examine the Australian stock market which has June tax year-end. They find both a December-January and July-August seasonal patterns[1]. Dimson and Marsh (1986) find that the size effect in the U. K. is very significant. Literature does not provide a conclusive justification for either firm size effect or seasonal effect but, as evident from the few of the cited studies, existence of size effect and seasonal effect is well documented. Laxmoureux and Sanger (1989) find that a firm size effect is not restricted to January only and average excess retunes are explained by logarithm of firm size. Their tests account for both heteroscedasticity and autocorrelation. Fama and French (1992) conducted a research using data of about fifty years and concluded that beta does not predict the stock market returns and hence, it cannot be salvaged. This conclusion is contradicting to most widely held and used model in finance, CAPM. This paper is an attempt to show that the conclusion arrived by Fama and French (1992) is a result of over aggregation. In this paper portfolios are constructed based on size and then beta will be used to factor in the size effect analysis. Bhardwaj and Brooks (1993) included market conditions into the analysis and find much smaller January effects when beta is allowed to vary in bull and bear markets. This study will incorporate all these three factors delineated in the few studies mentioned above, namely equity beta, firm size, and market conditions. This study is based on data from US markets only.

[1] Tax laws in Australia and USA are similar but Australian tax year ends in June.

3. Data and Methodology

Data

Following the methodology used by Fama and French (1992) firms listed on NYSE, AMEX and reported on NASDAQ are used for the analysis. Firms with missing stock prices in last ten days of June are excluded from the analysis. Firms that did not have valid returns for at least 24 months before June of year t are excluded from analysis. Firms without Standard Industrial Classification (SIC) codes are excluded. After imposing these three conditions we obtain 103,002 observations of which 62,487 are from NYSE and AMEX firms covering a period of 28½ years and the remaining 40,517 observations are from NASDAQ firms covering 18 years. These observations are then classified into ten size based portfolios for each year. Basic data and methodology of this study is very close to that of Fama and French (1992). This study improves the theory by incorporating the market conditions (bull and Bear) in the model.

Portfolio Construction

Following the methodology of Fama and French (1992) market value of each firm is calculated on June of year t, for avoiding the documented season effect (December-January) in stock prices. Every year in June, the break points for the ten size based portfolios are determined based on the market value of the stocks listed on NYSE. Based on those break points, stocks of NYSE, AMEX and NASDAQ are assigned to these ten portfolios. This is to make sure that each portfolio includes equal number of shares from NYSE. In order to reduce the documented correlation between size and beta (see Jegadeesh, 1992), each decile is further divided into ten portfolios on the basis of security beta called pre-ranking betas. Pre-ranking beta is calculated by using monthly data of 60 months, (a minimum of 24 months). Data from June 1962 to June 1964 is used only for calculating pre-ranking betas. In this study, we will estimate beta as the sum of the slopes in the time-series regression of portfolio return on the current and prior month's market return as suggested by Dimson (1979).

$$R_{p\,t} = \alpha + \beta_1 R_{m\,t} + \beta_2 R_{m,\,t-1} + \epsilon_{p\,t} \qquad (1)$$

$$\hat{\beta}_{E\,p} = \hat{\beta}_1 + \hat{\beta}_2 ,$$

where, $R_{p\,t}$ is the return on size-beta portfolio p in month t; $R_{m\,t}$ and $R_{m,\,t-1}$ are the returns on the equity market portfolio in time period t and t-1, respectively; $\hat{\beta}_{E\,p}$ is the equity beta for size-beta portfolio p; $\epsilon_{p\,t}$ is the random error term; and α, $\hat{\beta}_1$, and $\hat{\beta}_2$ are the regression coefficients. All pre- and post-ranking betas in this paper are calculated using the Dimson (1979) method. In these ten portfolios from each size based decile, the size of the firms is close, and betas are different. Using this methodology, a group of 100 size-beta based portfolios are created for each year. This procedure is repeated for each year of 28½ years. Appendix A and Appendix B will show summary information e.g., market value, number of firms, average monthly return and beta of these size-beta portfolio. From Appendix A we can see that number of securities vary from portfolio to portfolio and highest number of securities in any portfolio is 232 and lowest number of securities in any portfolio is 16. Appendix B, supports the firms size effect on raw data, gives an impression that portfolio consisting of smaller size firms have higher returns (comparing returns of second row with returns of eleventh row). Appendix B also gives an impression that the general relationship between portfolio beta and portfolio return is inverse for all rows, which is against the basic finance principle about risk and return. Any firm can move from one portfolio to another portfolio from year to year. It can be seen in Appendix C and Appendix D that firm sizes are close for each row, but betas increase as we go. Portfolio returns and portfolio betas are calculated for each month to arrive on a time series of 342 observations for each of the 100 portfolios. Using betas of each portfolio, expected returns are calculated by linear regression model. Four different proxies; Value-Weighted index, Equal-weighted index, Standard and Poor's 500 index, and NASDAQ Composite index are used for return on market.

Proposition 1: Average Returns are linearly related to a Portfolio's Equity Betas.

To test this proposition the following cross-sectional model is used:

$$R_{p\,t} = \gamma_{0\,t} + \gamma_{1\,t}\,\hat{\beta}_{Ep} + \epsilon_{p\,t} \qquad p = 1, 2, 3, \ldots, 100 \quad (2)$$

where, $R_{p\,t}$ is the return from size-beta portfolio p in the month t; $\hat{\beta}_{Ep}$ is the equity beta for size-beta portfolio p in the post-ranking period; γ_0 and γ_1 are the regression coefficients; and $\epsilon_{p\,t}$ is the random error term. In this model $\gamma_{1\,t}$ is the price, or risk premia, of the equity beta in month t. The *t*-statistics for the average monthly estimated γ_1 will be computed to determine statistical significance of $\hat{\gamma}_1$.

The first row of Appendix C presents average parameter estimates from cross-sectional regressions of size-beta portfolio average returns on equity betas when the data for total time period is analyzed. The coefficients, γ_0 and γ_1, are regression constant and the risk premia for the equity beta, respectively. As proposed, risk premia for equity beta is positive using all four proxies for market return, evidence of positive relationship with average return and beta. But *t*-statistics for γ_1 using all four market proxies are insignificant. So, initial results show that equity beta is not priced when examining the total sample period of 1964-1992. Fama and French (1992) also arrived on the same result when they analyzed data from 1963 to 1990, and before presenting the results contrary to most of the financial research, they extended the time period of analysis to half-century from 1941-1990, and arrived on the same conclusion that beta does not describe the average stock returns.

Proposition 2: Securities are priced differently in different Market Conditions.

Following the methodology of Bhardwaj and Brooks (1993), we incorporated market conditions into the analysis. The period from 1964 to 1992 was divided in two parts, bull markets and bear markets. Bhardwaj and Brooks (1993) and Fabozzi and Francis (1977) define bull months when return on market is greater than zero and bear market when return on market is less than or equal to zero. We use the Weisenberger Investment Companies Guide classification in which a bull (bear) market is defined as a time period during which the stock market increases (decreases) by at least ten percent from its most recent low (high). Appendix D lists the market conditions from July 1964 to December 1992. Out of total 342 month, for 233 months market was bullish and remaining 109 months market was bearish.

The same model (2) run again for these two parts (bear markets and bull markets) separately. The second row of Appendix A presents average parameter estimates from cross-sectional regressions of size-beta portfolio average return on equity beta for bear market conditions. The third row of Appendix A presents average parameter estimates from cross-sectional regressions of size-beta portfolio average return on equity beta for bull market conditions. Note, the following four observation in Appendix C:

1. The risk premia for equity beta (γ_1) for total time period and for all four return on market proxies is positive, and all of them are insignificant.
2. The risk premia for equity beta (γ_1) for total time period and for all four return on market proxies is positive but very close to zero.
3. The risk premia for equity beta (γ_1) for bear market conditions and all four return on market proxies is negative, and all of them are significant
4. The risk premia for equity beta (γ_1) for bull market conditions and all four return on market proxies is positive, and all of them are significant

Based on first and second observation, we reject the first proposition about the relationship between equity beta and returns. This conclusion is similar to Fama and French (1992) conclusion that beta does not describe the average stock returns. But, third and fourth observations make us draw the conclusion that risk premia for equity beta during bear market and bull market has negative and positive signs, respectively. Both risk premia for equity betas are statistically significant. This result is different from the result of first proposition. Hence, it shows that the risk premia for equity beta and returns shows a relationship. If markets are going up the risk premia for equity beta is positive, and if markets are going down the risk premia for equity beta is negative, which fits perfectly with investors' intuition. This risk premia whether market is bullish or bearish is significant. This result which is contradicting to the Proposition 1 is the result of analyzing the data for different market conditions separately. Still, we cannot conclude that the securities are priced differently during different market conditions, until we show that the two risk premium are significantly different from each other. For that we must show that the bear market risk premia for equity beta is statistically different from bull market risk premia for equity beta.

To test that the two risk premia for equity beta for bull market condition is different from risk premia for equity beta for bear market, *t*-statistics is calculated using the following equations:

$$t = \frac{\overline{\gamma}_{bull} - \overline{\gamma}_{bear}}{S_p \sqrt{\frac{1}{n_{bull}} + \frac{1}{n_{bear}}}} \qquad (3)$$

where, $$S_p = \sqrt{\frac{(n_{bull} - 1)\, S_{bull}^2 + (n_{bear} - 1) S_{bear}^2}{n_{bull} + n_{bear} + 2}}$$

where, S_p is the pooled standard deviation; S_{bull} is the standard deviation of observations in the bull market period; S_{bear} is the standard deviation of observations in the bear market period; n_{bull} is the number of months in bull market period; n_{bear} is the number of months in bear market period; $\bar{\gamma}_{bull}$ is the average monthly estimates during bull market period: and $\bar{\gamma}_{bear}$ is the average monthly estimates during bear market period. Results are shown in Table I.

Table 1: The *T*-Statistics for Comparing Equity Beta Risk Premia for Bear and Bull Periods Based on the Equation

$R_{pt} = \gamma_{0t} + \gamma_{1t}\,\hat{\beta}_{Ep} + \epsilon_{pt}$	
Proxy used for Market Returns	*t*-Statistics
Value-Weighted	5.6731
Equal-Weighted	4.8425
S&P 500	5.8162
NASDAQ Composite	4.9144

All these *t*-statistics are significant, showing that the risk premia for equity beta during bear market condition is significantly different from the risk premia for equity beta during bull market conditions. Hence, the second proposition is accepted that the securities are priced differently during different market conditions.

The result of first proposition is like conclusions arrived by Fama and French (1992), that beta does not describe the stock market returns. But the results of second proposition is contradicting to the results of Fama and French (1992) in two different ways. Firstly, beta is significant descriptor of stock market returns and secondly, the risk premia for equity beta during bull and bear markets is different. Risk premia for equity beta during bull market condition is positive and it is negative during bear market conditions.

4. Conclusion and Implications

In this paper two propositions were used to examine the relationship between equity beta and average return. First proposition measures the relationship between equity beta and average return and did not find significant relationship between the two variables. This conclusion is same as the conclusion arrived by Fama and French (1992). The second proposition was tested on divided data set on the basis of market conditions (bear markets and bull markets). Application of the same model on divided data set, resulted in showing strong relationship between equity beta and average return. The risk premia for equity beta in bull market conditions is positive and significant. The risk premia for equity beta during bear market conditions is negative and significant. These two risk premia for equity beta (positive and negative) are significantly different. These results indicate that the pricing effects of equity betas changes overtime depending on different stock market conditions. This conclusion is a significant contribution to existing body of literature.

Aggregation over lengthy time period may obscure the differential pricing existing within each market period and results will be misleading, like the one reached about beta by many afore mentioned researchers. Now future research is needed to test this conclusion about differential pricing. One of the limitations of this study is the data used. Data from United States of America is analyzed. Hence, this study should be replicated using recent data sets and can be replicated in different countries.

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APPENDIX A:

This appendix presents the average market value in millions of sampled firms with number of firms in each portfolio.
July 1964 – December 1992

This appendix presents the average market value of the firm (in millions) and number of securities and included in each size-beta portfolio.

		ALL	β_{E1}	β_{E2}	β_{E3}	β_{E4}	β_{E5}	β_{E6}	β_{E7}	β_{E8}	β_{E9}	β_{E10}
ALL	M. Value	527.94	473.35	835.73	601.04	564.47	523.02	558.33	530.99	459.71	426.01	313.99
	# of Firms	3338	425	366	341	316	306	301	294	303	298	388
MV1	M. Value	13.23	11.81	13.56	14.23	13.58	13.24	14.13	13.75	13.59	13.47	12.47
	# of Firms	1432	232	174	144	124	113	114	114	123	112	182
MV2	M. Value	41.36	36.22	41.07	42.27	41.84	41.88	42.19	41.36	44.06	40.21	42.81
	# of Firms	264	29	27	26	24	25	24	24	25	26	34
MV3	M. Value	72.14	62.60	65.29	71.86	74.76	73.10	75.29	73.33	74.07	74.39	77.06
	# of Firms	293	31	29	29	29	30	29	28	28	28	32
MV4	M. Value	114.14	103.12	115.86	120.18	121.02	116.00	114.69	111.08	111.68	115.89	110.86
	# of Firms	244	25	26	28	26	24	23	21	21	24	26
MV5	M. Value	174.16	158.88	170.24	176.34	179.93	181.23	172.32	174.22	172.85	176.91	178.06
	# of Firms	229	22	24	24	23	24	23	22	21	22	24
MV6	M. Value	267.94	246.06	257.73	279.77	276.81	276.38	267.13	267.16	261.04	272.74	270.30
	# of Firms	196	18	19	22	21	20	20	18	18	19	21
MV7	M. Value	424.24	392.51	404.82	421.50	431.14	436.72	432.40	436.33	424.05	428.85	432.40
	# of Firms	184	18	18	18	19	19	18	18	18	18	20
MV8	M. Value	709.07	678.67	715.78	713.48	700.29	705.92	724.51	709.54	682.11	730.77	729.68
	# of Firms	170	16	17	17	17	18	17	17	17	17	17
MV9	M. Value	1320.19	1308.24	1285.25	1314.54	1310.59	1320.75	1340.07	1327.14	1296.14	1328.48	1372.43
	# of Firms	165	17	16	17	17	17	17	16	16	16	16
MV10	M. Value	7221.52	8702.29	15326.16	8935.27	7303.03	6108.61	6672.21	6108.83	5193.09	4181.80	3699.46
	# of Firms	161	17	16	16	16	16	16	16	16	16	16

Appendix B:

This appendix presents the average monthly returns (in percent) and post-ranking betas of each size-beta portfolio. The post-ranking betas are calculated by using the Value-weighted Index of NYSE AMEX index for market portfolio. Appendix for post-ranking betas calculated using equal-weighted index, S&P 500 index and NASDAQ composite index are available at request.

July 1964 – December 1992

		ALL	β_{E1}	β_{E2}	β_{E3}	β_{E4}	β_{E5}	β_{E6}	β_{E7}	β_{E8}	β_{E9}	β_{E10}
ALL	**Return**		1.56	1.47	1.50	1.45	1.50	1.43	1.36	1.43	1.20	1.13
	Post Ranking beta		0.952	1.017	1.163	1.232	1.293	1.356	1.457	1.505	1.610	1.778
MV1	**Return**	1.62	1.87	1.64	1.68	1.62	1.73	1.69	1.45	1.75	1.28	1.34
	Post Ranking beta	1.488	1.080	1.197	1.374	1.403	1.488	1.566	1.677	1.697	1.802	1.875
MV2	**Return**	1.41	1.38	1.52	1.41	1.57	1.71	1.27	1.42	1.38	1.35	1.16
	Post Ranking beta	1.385	0.849	1.022	1.158	1.298	1.331	1.310	1.580	1.672	1.678	1.931
MV3	**Return**	1.30	1.14	1.42	1.52	1.42	1.36	1.40	1.52	1.55	1.05	0.73
	Post Ranking beta	1.340	0.822	0.958	1.117	1.243	1.333	1.349	1.473	1.570	1.695	1.786
MV4	**Return**	1.39	1.24	1.34	1.56	1.43	1.65	1.44	1.53	1.39	1.30	1.01
	Post Ranking beta	1.340	0.899	1.005	1.126	1.208	1.312	1.409	1.416	1.443	1.681	1.940
MV5	**Return**	1.28	1.41	1.45	1.38	1.29	1.25	1.34	1.49	1.17	1.08	0.97
	Post Ranking beta	1.281	0.761	0.938	1.064	1.095	1.203	1.354	1.434	1.460	1.661	1.819
MV6	**Return**	1.30	1.07	1.49	1.36	1.56	1.23	1.38	1.21	1.18	1.33	1.14
	Post Ranking beta	1.215	0.598	0.813	1.054	1.200	1.223	1.212	1.327	1.408	1.583	1.626
MV7	**Return**	1.15	1.11	1.30	1.28	1.24	1.31	1.16	1.10	1.01	1.07	0.94
	Post Ranking beta	1.194	0.619	0.889	1.025	1.111	1.178	1.257	1.332	1.352	1.426	1.709
MV8	**Return**	1.16	1.14	1.17	1.42	1.24	1.27	1.19	1.09	1.01	1.18	0.91
	Post Ranking beta	1.158	0.622	0.809	1.047	1.105	1.127	1.202	1.203	1.350	1.402	1.667
MV9	**Return**	1.08	0.93	0.95	1.13	1.19	1.23	1.29	1.11	0.96	1.09	0.92
	Post Ranking beta	1.056	0.594	0.741	0.960	0.991	1.057	1.172	1.166	1.211	1.192	1.455
MV10	**Return**	0.88	1.01	0.92	0.97	0.89	1.03	0.79	0.98	0.83	0.74	0.67
	Post Ranking beta	0.952	0.650	0.723	0.739	0.905	0.958	0.982	1.015	1.049	1.148	1.361

Appendix C:

Average Parametric Estimates from Month-By-Month Regressions of Size-Beta Portfoilo Returns on Portfolio's Equity Betas, All Firms, JULY 1964 – DECEMBER 1994[2]

This appendix table presents average parameter estimates from a cross-sectional regression of size-beta portfolio returns on equity betas. Results are presented for four different equity market proxies. The value-weighted and the equal-weighted market returns are with dividends, whereas S&P500 and NASDAQ are without dividends. The mean of γ's is the time-series average of parameter estimates from monthly cross-sectional regressions, and the standards deviation is the time-series standard deviation of parameter estimates from monthly cross-sectional regressions. The *t*-statistics is the mean estimate divided by its time series standard error. Results are presented for the total sample period, bull market and bear market, respectively.

$$R_{pt} = \gamma_{0t} + \gamma_{1t}\,\hat{\beta}_{Ep} + \epsilon_{pt}$$

		VALUE-WEIGHTED		EQUAL-WEIGHTED		S&P 500		NASDAQ COMPOSITE	
		γ_0	γ_1	γ_0	γ_1	γ_0	γ_1	γ_0	γ_1
TOTAL TIME PERIOD (342 months)	MEAN	0.0097	0.0021	0.0088	0.0041	0.0100	0.0019	0.0126	0.0050
	S. D.	0.0416	0.0571	0.0390	0.0652	0.0419	0.0577	0.0371	0.0666
	t (Mean)	4.3112	0.6939	4.1485	1.1631	4.4355	0.5948	4.9912	1.1119
BEAR MARKET (109 months)	MEAN	0.0034	-0.0224	-0.0068	-0.0201	0.0050	-0.0235	0.0035	-0.0223
	S. D.	0.0356	0.0555	0.0328	0.0629	0.0361	0.0560	0.0295	0.0645
	t (Mean)	0.9878	-4.2109	-2.1499	-3.3294	1.4329	-4.3753	0.8875	-2.5843
BULL MARKET (233 months)	MEAN	0.0127	0.0136	0.0160	0.0154	0.0124	0.0137	0.0158	0.0146
	S. D.	0.0439	0.0543	0.0396	0.0633	0.0441	0.0547	0.0391	0.0648
	t (Mean)	4.4033	3.8257	6.1658	3.7185	4.2924	3.8234	5.1183	2.8485

[2] The NASDAQ time period covered is from July 1975 to December 1992 (210 months). The market was bullish for 160 months and was bearish for 50 months.

Appendix D:

Stock Market Conditions

Classification of bull and bear markets are taken from Wiesenberger (1964-1993). Wiesenberger defines a bull (bear) market as a time period during which the market increases (decreases) by at least ten percent from its most recent low (high).

BULL MARKETS	BEAR MARKETS
July 1964 to April 1965	May 1965 to June 1965
July 1965 to January 1966	February 1966 to September 1966
October 1966 to December 1967	January 1968 to February 1968
March 1968 to November 1968	December 1968 to July 1969
August 1969 to October 1969	November 1969 to May 1970
June 1970 to April 1971	May 1971 to November 1971
December 1971 to December 1972	January 1973 to August 1973
September 1973 to October 1973	November 1973 to September 1974
October 1974 to June 1975	July 1975 to September 1975
October 1975 to December 1976	January 1977 to February 1978
March 1978 to August 1978	September 1978 to October 1978
November 1978 to September 1979	October 1979 to March 1980
April 1980 to December 1980	January 1981 to February 1982
March 1982 to September 1983	October 1983 to July 1984
August 1984 to August 1987	September 1987 to November 1987
December 1987 to June 1990	July 1990 to October 1990
November 1990 to December 1990	