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Damaged Durable Goods: Mitigating the Time Inconsistency Problem

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In this paper, we study the time inconsistency problem in the durable goods market with entry of new consumers in each period. The monopolist only wants to sell the product to high valuation consumers at the beginning. However, the accumulation of low valuation consumers makes it irresistible to hold a sale, which takes away the monopolist's profits from high valuation consumers. He can strategically introduce a damaged good and sell it to low valuation consumers to mitigate this problem. In a two period model, he sells the damaged good in each period. In an infinite horizon model, he introduces the damaged good with some delay. The social welfare is lower with the introduction of the damaged good than otherwise.

JEL Codes: D0, L0

1. Introduction

This paper studies the versioning in a durable good market. We show that the monopolist producer uses versioning to mitigate the time inconsistency problem. More specifically, he intentionally creates a downgraded version of the original good, or a "damaged good" to "clear" the market. In this way, he could charge a higher price on the original good and make more profits on the high valuation consumers.

The motivation of this paper comes from the popular versioning in our economy, especially in the electronics market. Many products are offered in different versions at different prices even when the costs are the similar. Flash drives are a good example. A 32GB flash drive costs twice as much as a 16GB flash drive although the cost of producing them is almost the same. Sometimes, it is even more costly to produce a "cheaper" product. Producers have to pay extra costs to take away part of the functions and sell a degraded version of the original product. In the last century, IBM damaged some functions of IBM Laser Printer to produce IBM Laser printer E, which was slower (5 pages per minute v.s 10 pages per minute) but much cheaper than the original one. Intel disabled the math coprocessor on 486DX microprocessor to create 486SX and sold it at a low price. Microsoft introduced Windows XP home edition after its professional edition[i]. For the newest Windows 7, it is said that "So…for Windows 7, we are using a single image for all SKUs. This means the bits for all the editions are already on your computer if you are running Windows 7. With Windows Anytime Upgrade, users can unlock and upgrade to a different SKU much easier than before."(LeBlanc

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2009) It is obvious that Microsoft had to pay extra cost to “lock” many functions of the Ultimate edition to create the Home Edition and sell it for $100 cheaper. To explain this phenomenon, Deneckere and McAfee (1996) first introduced the concept of “damaged goods". They argued that the monopolist produced damaged goods to do price discrimination and this could be a Pareto improvement under some parameter assumptions. However, in the electronics market, price discrimination may not be the only reason to explain versioning. Electronics are usually durable goods and firms are in a dynamic environment. Producers might use versioning to mitigate the well-known time inconsistency problem.

Coase (1972) first studied the durable goods monopolists’ time inconsistency problem. When consumers have different valuations on the durable goods, the monopolist uses intertemporal price discrimination to maximize his profit. First, he sets a high price *p* and sells the product to those with a valuation higher than *p*. Since the good is “durable”, consumers can use it for more than one period. They do not come back to the market next period. There are only consumers with valuation lower than *p*. The monopolist will do another maximization and set a price $p'<p$, to induce lower valuation consumers to buy at *p’*. As long as the price is higher than the marginal cost, he will keep on doing this to extract as much surplus as possible from consumers with different valuations. Unfortunately, high valuation consumers can anticipate what will happen after they buy the product and do not want to pay p in the first place. They will wait for the price cut and enjoy more surplus. To induce high valuation consumers to buy early, the monopolist has to lower the price. If the time interval between price changes goes to zero and consumers’ cost of waiting becomes negligible, the monopolist loses all his market power and has to set his price at the marginal cost.

The loss of profits prompts the monopolist to find ways to mitigate the time inconsistency problem. To focus on this problem, we intentionally set up a model in which a monopolist will not produce damaged goods in a static setting. We then study the need and timing to introduce damaged goods in a dynamic environment. In our model, the monopolist sells an infinitely durable good. New consumers enter the market every period. Each generation is identical, with both high valuation consumers and low valuation consumers in it. The measure of high valuation consumers in each generation is so high that the monopolist never wants to sell the durable good to low valuation consumers in a static model. In this way, our model is different from Deneckere and McAfee (1996). In the dynamic model in which consumers live forever, low valuation consumers would accumulate in the market. In a certain period *J*[ii], the temptation to sell the good to low valuation becomes irresistible and the monopolist will lower the price and hold a "sale" (Conlisk, Gerstner & Sobel 1984). We will refer to it as the CGS model hereafter. The sale creates a similar time inconsistency problem as in the traditional literature. To mitigate the problem, the monopolist can produce some damaged good and sell it to low valuation consumers. When the quality is low enough, the introduction of damaged goods can help the monopolist achieve higher profit than holding a sale in period nJ, where n is a positive integer. In reality, damaged goods are usually introduced into the market after the original goods. For example, paper cover books always come after the hard cover copies. In our paper, we find the rationale for the monopolist to introduce the damaged good with some delay.

Besides the difference in the model from Deneckere and McAfee (1996), our paper is also different from some previous papers on damaged goods, such as Takeyama (2002), Lee (2003) and Hahn (2006). The first two papers use a two-period model, in which the “end of the world” effect plays a role. Such an effect is not in our infinite horizon model. Hahn (2006) has only one generation that lives forever. We have new consumers in each period. The condition for the monopolist to sell the damaged good in the first period is less stringent in our paper.

Although damaged goods can mitigate the time inconsistency problem, it is always undesirable for the social welfare. Given that the marginal costs of producing the original good and the damaged good are the same, the first best result should be that all consumers buy the original good.

This paper is organized in the following way. The next section talks about the related literature. Section 3 sets up the model. Section 4 is the analysis and results. We first study the two period model and then generalize it to an infinite horizon model. Section 5 is the welfare analysis. Section 6 concludes the paper.

2. Literature Review

This paper is related to the strain of literature on durable goods market (See Waldman 2003 for a very good survey on this literature). First, it is related to papers on the time inconsistency problem, including Coase (1972), Stokey (1979, 1981), Bulow (1982), CGS model, Besanko and Winston (1990), Waldman (1993,1996), etc. Coase (1972) is the seminal paper in this area. However, it is Stokey(1979, 1981) and Bulow (1982) that prove Coase’s conjecture. CGS model studies the time inconsistency problem with the entry of new consumers every period. The main finding is that the price is cyclic. The monopolist is in a world with two types of consumers, high valuation and low valuation consumers. For a long time, he only sells the durable good to high valuation consumers. Low valuation consumers accumulate and in a finite period, *J,* say, the temptation to sell to low valuation consumers becomes irresistible. He lowers the price and clears the market. Again, high valuation consumers have rational expectation and demand enough surplus to buy the durable good right after they enter the market instead of waiting for the price cut. As it is closer to *J*, the monopolist needs to leave more surplus to them since their cost of waiting becomes smaller. Thus, the price is declining over time and reaches the lowest point in period *J*. In period *J+1*, a new cycle begins. Our paper is based on the finding in CGS model. We prove that the monopolist gain more profits from selling damaged goods and after the introduction of the damaged good, the price becomes constant. Some papers suggest renting as the mitigation of the time inconsistency problem. By renting out the durable good, the monopolist still has the ownership. He will internalize his pricing effect and not do anything that lowers the value of the durable good (Coase 1972, Bulow 1982, Fudenberg & Tirole 1998, Waldman 1996, etc.). However, renting is either prohibited by law (such as the 1953 United Shoe case) or not applicable in some cases: we may rent cars, but we rarely rent digital cameras, or laptops.

Second, it is related to damaged goods literature, such as Deneckere and McAfee (1996), Takeyama (2002), Lee (2003) and Hahn (2006). Takeyama (2002) studies a two period model and shows that the monopolist could strategically use vertical differentiation to mitigate the time inconsistency problem. The idea is the following: after high valuation consumers leave the market, the monopolist has the incentive to upgrade the low quality good to a high quality with some costs. The monopolist can strategically produce a low quality good such that low valuation consumers' value from upgrade is less than the upgrade cost. In this way, the monopolist makes a commitment that he will not upgrade the low quality good in the second period. Lee (2003) also considers the durable good monopolist's strategic use of damaged goods with a two period model. Unlike our paper, he does not allow new consumers in the second period and information is very important in his model. He finds out that the firm would produce a degraded good in the first period if consumers are semi anonymous (Fudenberg &Tirole 1998). None of the previous studies explained the phenomenon in an infinite horizon model.

Hahn (2006) is the most closely related to our paper. In his paper, there is only one generation of infinitely lived consumers. As for the introduction of a damaged good, the monopolist has three options: selling the damaged good in the first period and end the game; selling it with delay; not to introduce it at all. The decision depends on several parameters: the measure of high valuation consumers, the utility that high and low valuation consumers can get from the original good and the damaged good, and the cost of producing the durable goods. The welfare implication is ambiguous. If consumers are very patient, damaged good is welfare reducing.

Our paper is different from Hahn (2006) in several ways: First, there are new consumers in each period so that the game never ends. When the monopolist chooses his strategy, he always has to take into account its effect on his continuation payoff. The missing of the "end of the world" effect changes the analysis and the result. Second, in his paper, the condition for the first case, separating-without-delay, is the same as the one in the static model. That is, whenever it is optimal for the monopolist to do price discrimination in a one period static model, it is optimal for him to sell the damaged good in the first period of the dynamic game and end the game. Our paper does not need such a condition for the monopolist to sell the damaged good from the first period. Third, our welfare implication is unambiguous: damaged goods are always detrimental to the social welfare.

3. The Model

We adopt the overlapping generation model in CGS with discrete time, *t=1,2,3*....
Two types of consumers are infinitely lived: high valuation (H) and low valuation (L). Every consumer demands at most one unit of the durable good. High and low valuations consumers' life time utility from the good is $v_h \; and \; v_l, v_h > v_l > 0$, respectively. Their

reservation prices for the durable good are $v_h q$ and $v_l q$, where q is the quality of the durable good. H group has measure μ and L group has measure $1-\mu$. There is a continuum of homogeneous consumers in each group. A new cohort enters the market every period. We call them generation *x* if they arrive at period *x*. Every generation is identical.

An infinitely lived monopolist sells an infinitely durable good in the market. The quality is normalized to *1*. He can disable some functions of the original durable good to create a damaged good with quality $\theta < 1$. This process is costless. We can imagine that he just needs to change the code of a program, or install a different chip in a digital camera. Without loss of generality, we assume the marginal cost of producing both the original and the damaged goods is equal to 0. Due to some technology constraints, the quality of the damaged good θ is exogenously given θ might be the minimum standard of a product given by the administration. The monopolist can choose whether and when to introduce the damaged good into the market. However, the decision is irreversible. The damaged good cannot be withdrawn from the market. The monopolist could not make any binding commitments on future prices or quantity of the durable goods.

μ is common knowledge but the monopolist does not know the valuation of each consumer. As a result, he cannot do price discrimination based on consumers' types. To induce the high valuation consumers to buy the high quality good, he must give them enough incentive. Otherwise, they would be better off mimicking low valuation consumers. The monopolist and consumers discount the future at the same discount rate.

Although the durable good can last forever, we assume no secondhand market for it due to asymmetric information or adverse selection (Akerlof 1970). In the electronics market, purchasing goods in a secondhand market is a very risky decision because it is hard to know the exact quality of used electronics.

4. Analysis and Results

We first present some preliminary analysis.

We make the following assumption:

A.1 $\mu > \frac{v_l}{v_h}$

Under this condition, monopolists in a one-period static model would not sell the damaged good and those in Hahn (2006) would not sell the damaged good in the first period. In the two period model in our paper, the monopolist will sell the damaged good in every period if its quality is low enough. In the infinite horizon model, the monopolist will introduce the damaged good with some delay. To see the first argument, we need to know that as long as the monopolist wants to sell some good to the low valuation consumers, he will extract the entire surplus from them. Low valuation consumers are indifferent between buying and not buying. Furthermore, as long as he wants to sell some good to low valuation consumers, he will sell to them the same good that he sells to high valuation consumers. Let us assume that the monopolist has two versions, with the quality q_h and q_l. If he only sells to the high type, he can make a profit of $v_h q_h$. If he sells to both groups, he can only set the price of the high version at $p_h = v_h(q_h - q_l) + v_l q_l$. This comes from high valuation consumers' incentive constraint.

$$v_h q_h - p_h \geq v_h q_l - v_l q_l$$

where the price of the low version is $p_l = v_l q_l$. The right hand side is the surplus that high valuation consumers can get if they buy the low quality good.

The profit is

$$\pi = \mu[v_h(q_h - q_l] + v_l q_l] + (1 - \mu)v_l q_l$$

when $\mu > \frac{v_l}{v_h}$, we have the following result:

$$\mu v_h q_h - \mu[v_h(q_h - q_l] + v_l q_l] - (1 - \mu)v_l q_l$$
$$= \mu v_h q_l - v_l q_l$$
$$= q_l(\mu v_h - v_l) > 0$$

The monopolist has no incentive to discriminate. Instead, he sets a price equal to the high valuation consumers' reservation price and extracts all the surplus from them. If the monopolist sells the durable good with q_h to both groups, then the profit is

$$\pi_h = v_l q_h$$
$$\pi - \pi_h = \mu[v_h(q_h - q_l] + v_l q_l] + (1 - \mu)v_l q_l - v_l q_h$$
$$= (\mu v_h - v_l)(q_h - q_l)$$

which is negative if $\mu v_h < v_l$, or $\mu < \frac{v_l}{v_h}$. Notice that when $\mu < \frac{v_l}{v_h}$, only selling to high valuations consumers is not optimal either.

Next, we are going to show that in a dynamic model with durable goods, he faces a different problem. To see this, we can compare juice (nondurable goods) with camcorders (durable goods). Let's assume that both are sold by monopolists, A and B. If A's optimal policy is to sell to high valuation consumers only, he can stick to that policy because high valuation consumers will come back to the market very soon and the proportion of them would not change. A is in the same environment every period and his optimal strategy should be the same. B, on the other hand, faces different situations every period. After he sells the camcorders to high valuation consumers, they stay out of the market for a long time. He starts with a market with a high proportion of high valuation consumers and he only sells to these consumers. As time passes by, the proportion of low valuation consumers becomes higher and higher. At a certain time, when the proportion becomes high enough, selling to low valuation consumers becomes irresistible. CGS model proves that in a finite period J, the monopolist will lower the price of the original good to v_l and hold a "sale."

Rational consumers in the durable good market anticipate the "sale" and are only willing to pay $v_h - \delta^{J-t}(v_h - v_l)$ in period t, since if they wait until period J, they can secure themselves a surplus of $v_h - v_l$. As they come closer to J, the cost of waiting becomes smaller. Therefore, they are willing to pay a lower price in the current period. The price is declining over time. It reaches the lowest point in period J. Every consumer in the market buys the good and leaves the market. In $J+1$, a new cycle begins. The discounted sum of profit in this equilibrium is lower than the profit that the monopolist can get if he commits never to sell to low valuation consumers and charges a price of v_h every period. That is the well-known time inconsistency problem. It seems like the monopolist is competing with his future selves. This competition erodes his market power. Economists have found some ways to solve this problem. Renting seems to be an effective way (Coase1972, Stokey1979, 1981, Bulow1982, Fudenberg & Tirole 1998, etc.). By renting, instead of selling the durable goods, the monopolist still owns the durable good. He does not want to do something that lowers its value. Therefore, renting can completely solve the problem. Nevertheless, as what we said in the introduction, renting is either prohibited by law or not applicable in many markets.

This paper provides another way to mitigate the time inconsistency problem. The monopolist can produce a damaged good and strategically use price discrimination to remove or reduce the pressure to lower the price of the original good in the future. We start with the simplest case of a two period model and then generalize it to an infinite horizon model.

4.1Two Period Model

In this simplest model, new consumers enter the market in both periods. Generation 1 can make their purchases in period 1 or period 2, while generation 2 can only buy the durable good in period 2. Consumers try to maximize their surplus from the durable good by choosing which product (if there are more than one in the market) to buy and when (if they can choose) to buy it.

The monopolist's strategy is the products to offer in the market and how to price them. He wants to maximize his discounted sum of profits from the two periods. We use π to denote profits from some particular period(s) and Π to denote the overall lifetime profit.

4.1.1 The Commitment Case

Suppose for now the monopolist can make commitments on future prices and product lines. He will commit never to sell to low valuation consumers under assumption A.1. In this way, he is able to set the price of the durable good equal to the reservation price of high valuation consumers, v_h.

High valuation consumers do not need to wait for a sale so they make the purchase right after they enter the market. The profit for the monopolist comes from high valuation consumers in both periods

$$\Pi_c = \mu(v_h + \delta v_h)$$
$$= \mu v_h(1+\delta)$$

4.1.2 The Noncommitment Case

In the second period, the monopolist faces a different problem than the one in the first period. After the monopolist has sold the durable good to high valuation consumers, he would not care about the impact of his current strategy on those goods. Moreover, since high valuation consumers already bought the durable good and left the market, the environment in the second period is different from that in the first period. Only selling to high valuation consumers may not be optimal any more. The monopolist needs to adjust his production and pricing policy accordingly. We are interested in the subgame perfect Nash equilibrium in this case.

In the second period, as in a static model, the monopolist's decision of whether to sell the good to low valuation consumers depends on the measure of high valuation consumers. In the second period, there is a new generation with μ high valuation consumers and $1-\mu$ low valuation consumers. The proportion of high valuation consumers in the second period market is equal to

$$\gamma = \frac{\mu}{\mu + 2(1-\mu)}$$

The high valuation consumers in generation *1* already bought the durable good and left the market. The remaining $1-\mu$ low valuation consumers are still in the market.

Subcase 1 $\gamma > \frac{v_l}{v_h}$ or $\mu > \frac{2v_l}{v_h+v_l}$

As what happened in the first period, the monopolist does not want to sell to low valuation consumers. The price is still equal to v_h.The profit is the same as the commitment profit.

Subcase 2 □ $\gamma < \frac{v_l}{v_h}$ or $\mu < \frac{2v_l}{v_h+v_l}$

It is easy to show that the monopolist has the incentive to sell to low valuation consumers. He will lower the price to v_l. The profit from selling to both types is equal to $[\mu + 2(1-\mu)]v_l$ and the profit from selling only to the high type is equal to μv_h.

$$\mu v_h < [\mu + 2(1-\mu)]v_l$$

since

$$\frac{\mu}{\mu+2(1-\mu)} < \frac{v_l}{v_h}$$

High valuation consumers in the first period can anticipate this and are only willing to pay no more than $v_h - \delta(v_h - v_l)$ in the first period.

The discounted sum of profits is

$$\Pi_{nc} = \mu\big(v_h - \delta(v_h - v_l)\big) + \delta\big(\mu + 2(1-\mu)\big)v_l$$
$$= \mu v_h(1-\delta) + 2\delta v_l$$

which is smaller than Π_c given that $\mu > \frac{v_l}{v_h}$.

Lemma 1 If the monopolist cannot make commitments on the price and product lines, he will sell the original good to only high valuation consumers in the both period if $\mu > \frac{2v_l}{v_h+v_l}$*; if* $\frac{2v_l}{v_h+v_l} > \mu > \frac{v_l}{v_h}$*, he sells the good to high valuation consumers in the first period and to all consumers in the second period.*

The monopolist's profit is lower if he ever sells the good to low valuation consumers than if he only sells the good to high valuation consumers. He can make more profits if he can damage some part of the original good to produce a damaged good with quality θ□ and sell it to low valuation consumers in the first period. To show that introducing the damaged good in the first period is a subgame perfect Nash equilibrium, we use backward induction. If the monopolist introduces the damaged good in the first period, then he will sell it in both periods. If he does not introduce it, he will then decide whether to introduce it in the second period. According to our preliminary result, the monopolist will not introduce the damaged good. Either he only sells the original good to high valuation consumers or he sells it to both types, depending on the measure of high valuation consumers.

Knowing what will happen in the second period after the two histories---introducing or not introducing the damaged good in the first period, we will go back to the first period and find the monopolist's optimal decision.

Proposition 2 When $\mu < \frac{2v_l}{v_h+v_l}$*and* $\theta < \frac{2\delta}{1+\delta}$*, the monopolist sells the original good to high valuation consumers at a price* $v_h - (v_h - v_l)\theta$ *and the damaged good to low valuation consumers at a price* $v_l\theta$ *in both periods. The profit is equal to* $\Pi_{1d} = (1+\delta)[\mu v_h(1-\theta) + v_l\theta]$*. If* $\mu < \frac{2v_l}{v_h+v_l}$ *and* $\theta > \frac{2\delta}{1+\delta}$*, the monopolist will not introduce the damaged good. Instead, he will hold a sale in the second period.*

Proof: First, we look at the case when the monopolist did not sell anything to the low valuation consumers in the first period. If $\mu < \frac{2v_l}{v_h+v_l}$, then he will sell the original good to them in the second period at a price v_l. This is the no commitment case we have seen before. The profit is Π_{nc}. However, the monopolist can sell the damaged good to low valuation consumers in both periods at the price $v_l\theta$. At the same time, he can sell the original good to high valuation consumers at the price $v_h-(v_h-v_l)\theta,$ the total profit is $(1+\delta)[\mu v_h(1-\theta)+v_l\theta]$. The difference between this profit and Π_{nc} is

$$(1+\delta)[\mu v_h(1-\theta)+v_l\theta]-\mu v_h(1-\delta)-2\delta v_l$$
$$=(2\delta-\theta-\theta\delta)(\mu v_h-v_l)$$

which is positive if $\theta<\frac{2\delta}{1+\delta}$ and negative if $\theta>\frac{2\delta}{1+\delta}$. Done proof.

If the quality of the damaged good is relatively low, introducing the damaged good in the first period can help the monopolist mitigate the time inconsistency problem.

Interestingly, if the monopolist introduces the damaged good in the second period, not in the first period, his profit can be even bigger.

$$\Pi_{2d}=\mu[v_h-\delta(v_h-v_l)\theta]+\delta[\mu(v_h-(v_h-v_l)\theta)+2(1-\mu)v_l\theta]$$
$$=\mu v_h(1+\delta)-2\mu\delta(v_h-v_l)\theta+2(1-\mu)\delta v_l\theta$$

And

$$\Pi_{2d}-\Pi_{1d}=\mu v_h(1+\delta)-2\mu\delta(v_h-v_l)\theta+2(1-\mu)\delta v_l\theta-(1+\delta)[\mu v_h(1-\theta)+v_l\theta]$$
$$=(1-\delta)\theta(\mu v_h-v_l)>0$$

However, delaying the introduction of the damaged good to the second period is not the outcome of a subgame perfect Nash equilibrium. If he has not sold anything to the low valuation consumers, he will sell the original good, not the damaged good to them in the second period.

The two period model is just an illustration of the basic idea. It is far from being realistic. The major problem is that the second period is the last period. The monopolist's strategy would be different if there is a future after period *2*. He will consider the effect of his action on his future profits. That is what we will do next. It is also the main difference between our model and Hahn (2005). We will see that with a proper "punishment", we can support the subgame perfect equilibria in which the monopolist will introduce the damaged good in some finite period *t**.

4.2 Infinite Horizon Model

CGS model proves that the monopolist will hold a sale in period J. The total profit is equal to

$$\Pi_o = \frac{1}{1-\delta^J}\Big\{\mu\Big[\big(v_h - \delta^{J-1}(v_h - v_l)\big) + \delta\big(v_h - \delta^{J-2}(v_h - v_l)\big) + \cdots$$
$$+ \delta^{J-1}\big(v_h - (v_h - v_l)\big)\Big)\Big] + \delta^{J-1}J(1-\mu)v_l\}$$
$$= \frac{1}{1-\delta^J}[\mu\frac{v_h\,(1-\delta^J)}{1-\delta} - J\delta^{J-1}(\mu v_h\ - v_l)]$$
$$= \frac{\mu v_h}{1-\delta} - \frac{J\delta^{J-1}(\mu v_h\ - v_l)}{1-\delta^J}$$

If the monopolist sells a damaged good with quality θ in every period, his overall profit is given by

$$\Pi_d = \frac{1}{1-\delta}[\mu v_h\ - (\mu v_h\ - v_l)\theta] = \frac{\mu v_h}{1-\delta} - \frac{(\mu v_h\ - v_l)\theta}{1-\delta}$$

which is a decreasing function of θ since $\frac{(\mu v_h - v_l)}{1-\delta} > 0$ given our assumption on μ. There is no commitment problem if the monopolist sells the damaged good in every period. Π_d is an achievable profit. We can show that $\Pi_d > \Pi_o$ for some θ.

$$\Pi_d > \Pi_o$$
$$\Leftrightarrow \frac{J\delta^{J-1}}{1-\delta^J} > \frac{\theta}{1-\delta}$$
$$\Leftrightarrow \theta < \frac{J\delta^{J-1}(1-\delta)}{1-\delta^J}$$

CGS model also finds that J only depends on $\mu, v_h, v_l\ and\ \delta$. Thus the right hand side of the inequality is just a function of $\mu, v_h, v_l\ and\ \delta$.

When the inequality holds, the monopolist can at least do better by selling the damaged good every period than holding a sale in period J. This is a similar result as in our two period model.

The next thing we want to show is that the monopolist will actually introduce the damaged good with some delay. First, his profit is bigger if he introduces the damaged good in the period *2* than in period *1*.

$$\Pi_{1d} = \Pi_d$$
$$\Pi_{2d} = \mu(v_h - \delta(v_h - v_l)\theta + \delta(1-\mu)v_l\theta + \delta\Pi_d$$
$$\Pi_{1d} - \Pi_{2d} = (1-\delta)\Pi_d - \mu(v_h - \delta(v_h - v_l)\theta - \delta(1-\mu)v_l\theta = -(\mu v_h - v_l)\theta(1-\delta) < 0$$

When calculating Π_{2d}, we separated the profit from the low valuation consumers in the first and second generation. Although the low valuation consumers in the first generation purchased the damaged good in the second period, we treated them as if they purchase it in the first period. In this way, starting from the second period, the continuation profit is equal to Π_d. It is easier to compare Π_{2d} with Π_d.

We want to find the period t that the monopolist will introduce the damaged good.

$$\Pi_{td} - \Pi_{(t+1)d} = \mu(v_h - (v_h - v_l)\theta) + t(1-\mu)v_l\theta + \delta\Pi_d - \mu(v_h - \delta(v_h - v_l)\theta) \\ - \delta t(1-\mu)v_l\theta - \delta\Pi_d = -(\mu v_h - v_l)\theta(1-\delta) + (t-1)(1-\mu)v_l\theta(1-\delta)$$

Thus, $\Pi_{td} \geq \Pi_{(t+1)d} \Leftrightarrow t \geq \frac{\mu(v_h - v_l)}{(1-\mu)v_l}$

*t** would be the smallest integer that satisfies the above inequality. Using a similar argument as in CGS model, we can prove that in some period *T*, the monopolist would definitely sell the damaged good if he did not sell it before. Therefore we can find *t** recursively. At *T-1*, if the monopolist does not sell the damaged good, he knows he would do so in the next period. We compare the profit from selling the damaged good at *T-1* and the profit from selling it at *T*. If the former is bigger, he sells it at *T-1.* We do the same comparison between selling the damaged good at *T-2* and selling it at *T-1*, and so on.

In the equilibrium, the price of the original good is given by

$$p_t = v_h - \delta^{t^*-t}(v_h - v_l)\theta$$

from *t= 1* to *t**

The monopolist introduces the damaged in period *t** and the price is $p_d = v_l\theta$. High valuation consumers buy the original good in every period. Low valuation consumers wait until period *t** to buy the damaged good. After that, the monopolist sells the original good and damaged good in every period. The price of the original good is $p_o = v_h - (v_h - v_l)\theta$. High valuation consumers buy the original good and low valuation consumers buy the damaged good right after they enter the market. Many possible punishments that can support this as a subgame perfect Nash equilibrium. We will use a simple punishment in which the monopolist has to hold a sale every period after any deviation. The consumers will only accept a price no higher than v_l. The monopolist will not introduce the damaged good before *t** since $\Pi_{td} < \Pi_{t^*d}$ for all $t < t^*$. He will not introduce the damaged good later than *t** since $\pi_{t^*d} \geq \pi_{(t^*+1)d}$. He will also not hold a sale before or at period *t**[iii] because the possible gain is dominated by the loss in the continuation profit.

A summary of the results is provided in the following table:

Summary of Results

Two Period Model		High Valuation Consumers		Low Valuation Consumers		Profit
		Goods	Price	Goods	Price	$\mu v_h(1+\delta)$
	Commitment	Original	v_h	None	N/A	
	Non commitment					
	$\mu > \frac{2v_l}{v_h + v_l}$	Original	v_h	None	N/A	$\mu v_h(1+\delta)$
	$\mu < \frac{2v_l}{v_h+v_l}$ and $\theta < \frac{2\delta}{1+\delta}$	Original good	$v_h - \delta(v_h - v_l)$ in the first period and v_l in the second period.	Original good in the second period	v_l	$\mu v_h(1-\delta) + 2\delta v_l$
	$\mu < \frac{2v_l}{v_h+v_l}$ and $\theta > \frac{2\delta}{1+\delta}$	Original good	$v_h - (v_h - v_l)\theta$	Damaged good	$v_l\theta$	$(1+\delta)[\mu v_h(1-\theta) + v_l\theta]$
Infinite Horizon Model		Original good	$v_h - \delta^{t^*-t}(v_h - v_l)\theta$ from t= 1 to t* and $v_h - (v_h - v_l)\theta$ after t*	Damaged good after it is introduced in period t*	$v_l\theta$	$\sum_{t=1}^{t^*} \mu\, \delta^{t-1} v_h + t^*\delta^{t^*-1}\theta(v_l - \mu v_h) + \delta^{t^*}\frac{\mu v_h + \theta(v_l - \mu v_h)}{1-\delta}$

t* is the smallest integer that satisfies the inequality $t \geq \frac{\mu(v_h - v_l)}{(1-\mu)v_l}$.

4.3 Welfare Analysis

In this section, we want to compare the welfare in all three cases: (i)the monopolist can make commitments, (ii)hold a sale in period J and (iii)sell damaged goods in period *t**. By convention, the social welfare is equal to the sum of the monopolist's profit and consumers' surplus.

When the monopolist can make commitments, he will never sell anything to low valuation consumers and will charge a price equal to high valuation consumers' reservation price. The social welfare is just equal to his profit.

MA

$$W_c = \Pi_c = \frac{\mu v_h}{1-\delta}$$

When the monopolist cannot make commitments and has to hold a sale in period J, the social welfare is equal to the sum of his profit and the consumer surplus from high valuation consumers. Low valuation consumers' surplus is always equal to *0*.

$$W_J = \Pi + CS_h = \frac{1}{1-\delta^J}\left[\mu \frac{v_h\,(1-\delta^J)}{1-\delta} - J\delta^{J-1}(\mu v_h - v_l) + J\delta^{J-1}\mu(v_h - v_l)\right]$$
$$= \frac{\mu v_h}{1-\delta} + \frac{J\delta^{J-1} v_l (1-\mu)}{1-\delta^J} > W_c$$

High valuation consumers in each generation can get a surplus of $\mu\delta^{J-1}(v_h - v_l)$, discounted to the first period. This is just equal to the monopolist's profit loss from them. The monopolist can also make profits from low valuation consumers. Thus, the extra social welfare here is equal to the monopolist's profits from low valuation consumers.

If the monopolist can sell a damaged good with quality $\theta < \frac{J\delta^{J-1}(1-\delta)}{1-\delta^J}$, then in the equilibrium, he will sell it at period *t**. The social welfare is the following:

$$W_d = \Pi_{t*d} + CS_{hd} = \frac{\mu v_h}{1-\delta} + \delta^{t*-1} t^*(1-\mu) v_l \theta + \frac{\delta^{t*}}{1-\delta}(1-\mu) v_l \theta < W_J$$

To see why $W_d < W_J$, let's first compare the welfare if the monopolist sells the damaged good from the first period and W_J.

$$W_{1d} = \frac{\mu v_h}{1-\delta} + \frac{(1-\mu) v_l \theta}{1-\delta} < W_J$$

since $\theta < \frac{J\delta^{J-1}(1-\delta)}{1-\delta^J}$. Next, $W_{1d} > W_d$ since the profit from low valuation consumer is smaller if the monopolist delays the introduction of the damaged good. Starting from generation t^* his profit from low valuation consumers is the same in the two scenarios. However, his sales to generation *1* to *t*-1* low valuation consumers are all delayed thus the profit is smaller.

Now, we have the order of the welfare $W_c < W_d < W_J$. For policy suggestion, we would say that banning the damaged good is better for social welfare. Also, banning any binding contract which asks the monopolist to fix the price is also socially efficient.

5. Conclusion

In this paper, we adopt a similar model as CGS model and show that the monopolist can increase his profit by selling some damaged good. Unlike the existing papers on damaged goods, we study both a two period model and an infinite horizon model, with new consumers every period. In the two period model, we focus on the scenario when it is not optimal to do price discrimination in a static game, the monopolist will sell the damaged good in each period. In the infinite horizon model, there is no "end of the world" effect. The monopolist always has to consider how his strategy will affect the market in the future. He will delay the introduction of the damaged good. In terms of welfare, damaged good is harmful. Compare to the commitment case, noncommitment monopolist creates extra social surplus, which is exactly equal to his profit from low valuation consumers.

Endnotes

i Refer to Deneckere and McAfee (1996) for more examples, especially non- electronics.

ii The J here is actually the n* in CGS model.

iii The difference between holding a sale at some period t and sell the damaged good at period t has the same sign as $\frac{\mu(v_h-v_l)}{1-\delta}-(t-1)(1-\mu)v_l$. As long as $t<\frac{\frac{\mu(v_h-v_l)}{1-\delta}}{(1-\mu)v_l}+1$, the monopolist would not have the incentive to hold a sale. We know that t* is the smallest integer that satisfies $t\geq\frac{\mu(v_h-v_l)}{(1-\mu)v_l}$. Thus, $t^*<\frac{\mu(v_h-v_l)}{(1-\mu)v_l}+1<\frac{\frac{\mu(v_h-v_l)}{1-\delta}}{(1-\mu)v_l}+1$.

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