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Multi Product Multi Period Aggregate Production Planning in an Uncertain Environment

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In this paper a new approach is proposed to handle the uncertain input value in the aggregate production planning model.Generally aggregate production planning models have been developed by considering certain value of each parameter over a planning horizon. But in real life these values are not remain certain over a planning horizon rather uncertain.The main objective of this proposed appraoch is to find out the minimum total aggregate production planning cost considering uncertain input value. Hence, random value is considered from specific data range of each parameter of a specific automobile facory and finally compares the results with previous different approaches.

Keywords: Random value, Data range, imprecise parameters and uncertain environment.

1. Introduction

Demand forecasting can be classified as short, medium and longe range. Aggregate Production Planning (APP) is an intermediate based capacity planning and used medium range forecast, usually 3 to 18 months. The main attention of aggregate production planning is to determine production, inventory, and work force levels to meet fluctuating demand. The planning horizon is often divided into periods. Every month of a year may be considered one period. The form of time horizon for aggregate production planning varies from company to company. Generally aggregate production planning determines the best way to meet forecasted demand in the intermediate time by adjusting regular time production, overtime production, subcontracting levels, inventory levels, labor levels, backordering rates and other controllable variables.Theoretically APP controllable variables are assumed deterministic in nature but in real-world they are not deterministic. It is found that APP input parameters such as forcasted market demand, corresponding operating cost and capacity are uncertain in nature owing to some information being incomplete or unobtainable.

From past studies it is found that few authors have tried to solve the aggregate production planning problem considering imprecise data. To attain their goal they have used certain value for a specific time horizon and other researchers have used triangular fuzzy number for getting APP decision. From review guideline it is found that researchers mentioned that better aggregate production planning decision may

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be obtained if APP input data randomly consider in the model. But nobody has carried out research in such a way where APP input data randomly considered in their model. Hence the authors of this paper are highly motivated to fill up this research gap. This is the main cause for carry out the research work on this problem arena. The major distinguishable feature in this paper is that random value is to be considered randomly in the model. Then this random value is to be used both in the constraints and objective function iteratively for searching the minimum aggregate production planning cost. According to review considering uncertainty in APP model is very much important for produce more acceptable result. The major significance side of this new approach is that it incorporated uncertainty in the APP model. So, APP practioners get a clear guidline for handling uncertainty to solve their real life problem.

The rest of the paper is organized as follows: literature review is presented in section 2. Research methodology and mathematical statements are mentioned in section 3. Data collection and solving procedure are mentioned in section 4. Resuls and findings are presented in section 5. Section 6 and 7 represent conclusion and recommendation for further research respectively and finally the references are mentioned in the last portion of this paper.

2. Literature Review

Wang and Fang (2001) mentioned that Aggregate Production Planning (APP) is a medium range capacity planning. It typically encompasses a time horizon from 2 to 12 months. They mentioned that input data of APP are fuzzy in nature. But they considered crisp value and apply trapezoidal membership function to determine the APP result. Generally when APP input data are considered as a crip value then these data do not behave fuzzy nature rather deterministic in nature. Leung et al (2003) proposed a multi-objective model which is developed to solve the production planning problem for maximize profit and minimize workforce level. For practical implications of the proposed model different managerial production plans are evaluated. Forecasted demand and others operating costs are varied from time to time. This variation should be considered during making decision regarding on aggregate production planning but that are ignored in their research study. Pradenas et al (2004) mentioned that aggregate production planning is particularly complex in nature for producing different products. They considered demands and corresponding operating costs are deterministic in nature but in real life these data are uncertain in nature. Wang and Liang (2004) mentioned that traditional methods are used precise data to solve APP problem. But in real world to get appropriate APP result the input data are need to consider imprecise owing to some information is incomplete or unobtainable. Jana and Roy (2005) discussed multi-objective linear programming problem with fuzzy objective function and fuzzy constraints. For doing solution they considered triangular fuzzy numbers which are deterministic in nature. Wang and Liang (2005) mentioned that in real world to get appropriate APP result the input data need to consider imprecise. To attain this goal they considered triangular fuzzy numbers and then fuzzy logic is applied. Jain and Palekar (2005) mentioned that most of the research on aggregate production planning focused on discrete parts manufacturing. They mentioned that where intermediate inventory is not allowed and multiple products are produced simultaneously using complex configurations of production machines traditional APP approach may produce erroneous results. They proposed configuration-based model from resource-based model. But assumed deterministic input data for APP parameters which are not always deterministic in real life. Takey and Mesquita (2006) claimed that many industrial products are highly seasonally varied. To meet fluctuating demands make to stock strategy is used in terms of lowest inventory cost. But they considered fix operating costs which may variy with demand variation that is not considered in their study. Filho et al (2006) claimed that nobody integrate the objectives defined in manufacturing strategy in aggregate production planning model. Hence they proposed a multi objective model that included the context of manufacturing strategy. In order to accomplish their research works they considered demand and cost parameters are deterministic in nature. Jamalnia and Soukhakian (2009) mentioned that to get appropriate result from APP problems the input data need to consider imprecise. But it is noticed that they assigned certain value for every specific parameter. Gulsun et al (2009) mentioned that problems that involved a range data for a specific parameter need to take into account not only cost related objectives but also motivational or service level objectives simultaneously. They also mentioned that another important point is the uncertainty and flexibility factors. They mentioned that objective function of the model can be expressed with interval numbers. But it is noticed that in their work they assumed deterministic values of the APP parameters rather than interval value.

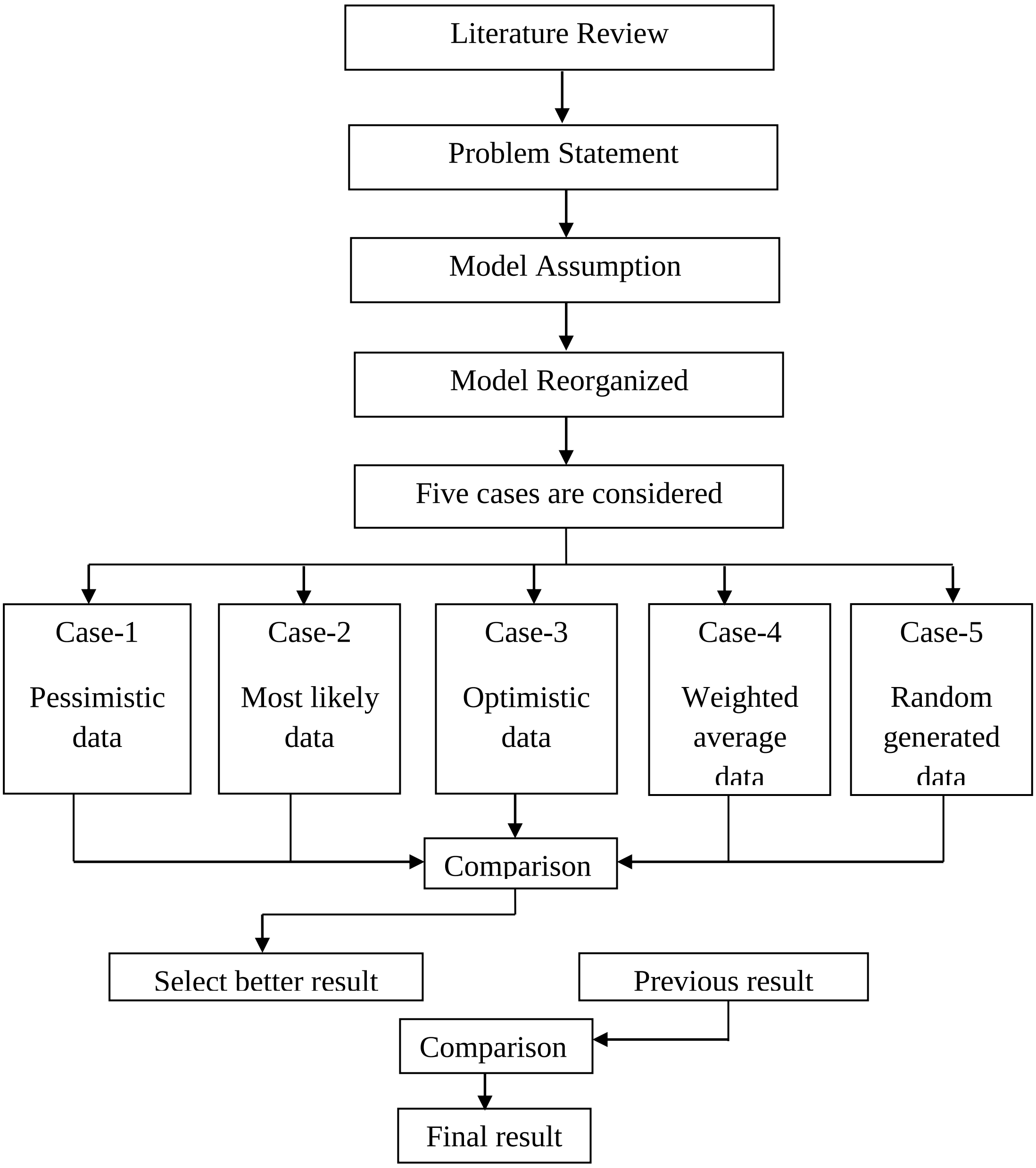
Baykasoglu and Gocken (2010) mentioned that in real world APP input data are imprecise because some information is incomplete or unobtainable. They consider triangular fuzzy numbe for some of the parameters and fix number for others. Effati and Abbasiyan (2010) focused on linear programming problems in which both the right-hand side and the technological coefficients are fuzzy numbers. They mentioned that according to Bellman and Zadeh (1970) approach obtained crisps values can be non-linear, where the non-linearity arises in constraints. They proposed two methods such as augmented lagrangian penalty method and method of feasible direction to solve this problem. Hashem et al (2011) mentioned that some critical parameters such as customer demand, corresponding operatin cost and manufacturing capacity are not known with certainty. So decision based on the certain value is a risk that demand might not be met with the right products. Mondal and Pathak (2011) mentioned that demand fluctuation is available in a dynamic market. Assigning a set of crisp values for solving APP problem is no longer appropriate. They applied fuzzy triangular member function and recommend that better result may be obtained by using random variable. Veeramani et al (2011) mentioned that fuzzy multi-ojective linear programming is one of the most frequently applied in fuzzy decision making techniques. They mentioned that objective functions and the constraints involve many parameters whose possible values may be assigned by the experts. In traditional approaches, such parameters are fixed at some values in an experimental or subjective manner throught the expert understands of the nature of the parameters. Unfortunately, real world situations are often not deterministic. There exist various types of uncertainities in social, industrial and economic systems. Yenradeea and Piyamanothorna (2011) mentioned that there is a link between marketing promotion and customer demand. Demand is always affected by marketing promotion. It is beneficial to determine both plans simultaneously. They proposed a general optimization model that can help making decision in marketing promotion in accordance with production planning in order to maximize the profit for the company. They assumed APP parameters are deterministic in natures which are not always deterministic. Ramezanian et al (2012) mentioned that aggregate production planning is a medium-term capacity planning to satisfy fluctuating demand over a planning horizon. In their paper they considered all programming input data deterministic in nature and there is no randomness. Huang and Chen (2012) mentioned that enterprises around the world have increasingly emphasized aggregate production planning for deciding an appropriate way to match many controllable factors.They recommend that to get proper aggregate production planning result uncertainty of input data should be considered. Chakrabortty and Hasin (2013) mentioned that market demand and curresponding operating cost and capacities are uncertain in real life. They considered triangular fuzzy number for solving APP problem. Madadi and Wong (2013) mentioned that aggregate production planning is a process by which a company determines ideal levels of capacity, production, subcontracting, inventory, stok out, and even pricing over a specified time horizon to meet customer’s requirements. In their study quality of products tried to consider and all input data are assumed deterministic during the time horizon. Mortezaei et al (2013) mentioned APP problems with deterministic parameters are unsuitable for determining effective solution. According to their views to get appropriate APP result forecasted demand, corresponding operating cost and capacities should be considered imprecise in nature.

Ning et al (2013) mentioned that for getting appropriate result of aggregate production planning all characteristics such as market demand, production cost and subcontracting cost must be considered uncertain in nature. Cheng and Bing (2013) mentioned that production planning and control has crucial impact on the production and business activities of enterprise.They proposed a multi-objective production planning optimization model based on the point of view of the integration of production planning and control, in order to achieve optimization and control of enterprise manufacturing management. They assumed all kinds of parameters such as demand, capacity and costs are deterministic in nature. Nowak (2013) mentioned that uncertainties are ignored in any managerial decisions but making sound decision uncertainty and risk of uncertainty have to be considered. Damghani and Shahrokh (2014) proposed a new multi-product multi-period multi-objective aggregate production planning model where they focused three objective functions, including minimizing total cost, maximizing customer services level, and maximizing the quality of end-product concurrently. Then, a fuzzy goal programming (FGP) is proposed to solve the proposed model. Forecasted demand and corresponding operating cost generally vary from period to period but these are not considered here. Sultana et al (2014) mentioned that aggregate planners are concerned with the quality and the timing of expected demand. They mentioned that the task of aggregate planner is to achieve equality of demand and capacity over the entire planning horizon. It is noticed that demand and operating costs are varied over the planning horizon. Chen and Huang (2014) mentioned that aggregate production planning plays a critical role in supply chain management. They investigated multi product, multi period APP problems with several distinct types of fuzzy uncertainties. In their work they assumed three possible values of the parameters and applied triangular membership function to get the solution. Throughtout the literature review it is found that most of the researchers are aggred to get appropriate aggregate production planning result input data should be considered as imprecise.

3. Research Methodology and Mathematical Statement

3.1 Research Methodoly

Outline of the research methodology is mentioned as follows:



Based on past studies limitations, data are newly assumed for generating better result. Five cases are considered as follows: Case 1: Pessimistic value, Case 2: Most likely value, Case 3: Optimistic values, Case 4: Pessimistic, Most likely and Optimistic values jointly used by applying weighted average method and finally Case 5 (Proposed approach): Random values are considered. Mathematical model is reorganized for adopting new data assumption in the current model. Accordingly secondary data have been adoted in the model and it is executed in application software matlab 2010 version for getting result. Finally compare the result with the previous results for taking decision.

3.2 Mathematical Statement

3.2.1 Current Mathematical Model

The multi-product multi-period APP problem can be described as follows. Assume that a company manufactures N types of products to satisfy the market demand over a planning horizon T. Based on the above characteristics of the considered APP problem, the mathematical model herein is developed on the following assumptions.

i. The values of all parameters are certain over the next T planning horizon.
ii. Actual labor levels, machine capacity and warehouse space in each period cannot exceed their respective maximum levels.
iii. The forecasted demand over a particular period can be either satisfied or backordered, but the backorder must be fulfilled in the next period.

The following notation is used after reviewing the literature and considering practical situations (Wang and Liang, 2004; Masud and Hwang, 1980; Wang and Fang, 2001)

N: Types of products
Z: Total costs ($)
D_{nt}: Forecasted demand for nth product in period t (units)
a_{nt}: Regular time production cost per unit for nth product in period t ($ /unit)
Q_{nt}: Regular time production volume for nth product in period t (units)
b_{nt}: Overtime production cost per unit for nth product in period t ($ /unit)
O_{nt}: Overtime production volume for nth product in period t (units)
c_{nt}: Subcontracting cost per unit of nth product in period t ($ /unit)
S_{nt}: Subcontracting volume for nth product in period t (units)
d_{nt}: Inventory carrying cost per unit of nth product in period t ($ /unit)
I_{nt}: Inventory level in period t for nth product (units)
e_{nt}: Backorder cost per unit of nth product in period t ($ /unit)
B_{nt}: Backorder level for nth product in period t (unit)
K_t: Cost to hire one worker in period t ($/man-hour)
H_t: Worker hired in period t (man-hour)
m_t: Cost to layoff one worker in period t ($/man-hour)
F_t: Workers laid off in period t (man-hour)
i_{nt}: Hours of labor per unit of nth product in period t (man-hour/unit)
r_{nt}: Hours of machine usage per unit of nth product in period t (machine-hour/unit)
V_{nt}: Warehouse spaces per unit of nth product in period t (ft^2/unit)
W_{tmax}: Maximum labor level available in period t (man-hour)
M_{tmax}: Maximum machine capacity available in period t (machine-hour)
V_{tmax}: Maximum warehouse space available in period t (ft^2)

$$\boldsymbol{Min\ Z} = \sum_{n=1}^{N}\sum_{t=1}^{T}[a_{nt}\,Q_{nt} + b_{nt}O_{nt} + c_{nt}S_{nt} + d_{nt}I_{nt} + e_{nt}B_{nt}] + \sum_{t=1}^{T}(K_tH_t + m_tF_t) \quad (1)$$

Here the first five terms are used to calculate production costs. The production costs include five components-regular time production, overtime, and subcontracts, carrying inventory and backordering cost. The later portion specifies the costs of change in labor levels, including the costs of hiring and lay off workers.

Constraints: Constraints on carrying inventory:

$$I_{n(t-1)} - B_{n(t-1)} + Q_{nt} + O_{nt} + S_{nt} - I_{nt} + B_{nt} = \breve{D}_{nt} \quad for\ \forall n, \forall t \qquad (2)$$

Where, $\widehat{D}_{nt}$ denotes the imprecise forecast demand of the nth product in period t. In real-world APP decision problems, the forecast demand $\widehat{D}_{nt}$ cannot be obtained precisely in a dynamic market. The sum of regular and overtime production, inventory levels, and subcontracting and backorder levels essentially should equal the market demand, as in first constraint Equation. Demand over a particular period can be either met or backordered, but a backorder must be fulfilled in the subsequent period.

Constraints on Labor levels:

$$\sum_{n=1}^{N} i_{n(t-1)} \left(Q_{n(t-1)} + O_{n(t-1)}\right) + H_t - F_t - \sum_{n=1}^{N} i_{nt} \left(Q_{nt} + O_{nt}\right) = 0 \quad for\ \forall t \qquad (3)$$

Here in the 3rd constraint equation represents a set of constraints in which the labor levels in period t equal the labor levels in period t-1 plus new hires less layoffs in period t.

Actual labor levels cannot exceed the maximum available labor levels in each period, as in 4th equation. Maximum available labor levels are imprecise, owing to uncertain labor market demand and supply.

$$\sum_{n=1}^{N} i_{nt} \left(Q_{nt} + O_{nt}\right) \leq W_{tmax} \quad for\ \forall t \qquad (4)$$

Constraints on Machine capacity:

$$\sum_{n=1}^{N} \hat{r}_{nt} \left(O_{nt} + Q_{nt}\right) \leq \widehat{M}_{tmax} \ \ for\ \forall t \qquad (5)$$

Constraints on Warehouse space:

$$\sum_{n=1}^{N} V_{nt} I_{nt} \leq V_{tmax} \quad for\ \forall t \qquad (6)$$

Here $\hat{r}_{nt}$ and $\widehat{M}_{tmax}$ denote the imprecise hours of machine usage per unit of the nth product and the imprecise maximum available machine capacity in period t, respectively. Above two equations represent the limits of actual machine and warehouse capacity in each period. Non-negativity Constraints on decision variables are:

$$Q_{nt}, O_{nt}, S_{nt}, I_{nt}, B_{nt}, H_t, F_t \geq 0 \ \ for\ \forall n, \forall t \qquad (7)$$

It is noted that current mathematical model will be used for Case 1, Case 2 and Case 3.

3.2.2 Improved Mathematical Model for Case 4

Model assumptions

i. Three values are considered for each of the parameter over the next T planning horizon.
ii. Actual labor levels, machine capacity and warehouse space in each period cannot exceed their respective maximum levels.
iii. The forecasted demand over a particular period can be either satisfied or backordered, but the backorder must be fulfilled in the next period.

Mathematical notation is same as like current model.

w_1: Weight of the pessimistic value
w_2: Weight of the most likely value
w_3: Weight of the optimistic value

$w_1 + w_2 + w_3 = 1$ and $w_1 = w_3 = \frac{2}{6}$ and $w_3 = \frac{4}{6}$ because for using the most likely values here is that the most possible values usually are the most important ones, and thus should be assigned more weights.

β = minimum acceptable possibility of converting imprecise value into precise value (0.5)

Accordingly improved objective function

$$\boldsymbol{Min\ Z} = \sum_{n=1}^{N}\sum_{t=1}^{T}\Big[\Big(w_1 a_{nt,\beta}^{p} + w_2 a_{nt,\beta}^{m} + w_3 a_{nt,\beta}^{o}\Big) Q_{nt} + \Big(w_1 b_{nt,\beta}^{p} + w_2 b_{nt,\beta}^{m} + w_3 b_{nt,\beta}^{o}\Big) O_{nt} + \Big(w_1 c_{nt,\beta}^{p} + w_2 c_{nt,\beta}^{m} + w_3 c_{nt,\beta}^{o}\Big) S_{nt} + \Big(w_1 d_{nt,\beta}^{p} + w_2 d_{nt,\beta}^{m} + w_3 d_{nt,\beta}^{o}\Big) I_{nt} + \Big(w_1 e_{nt,\beta}^{p} + w_2 e_{nt,\beta}^{m} + w_3 e_{nt,\beta}^{o}\Big) B_{nt}\Big] + \sum_{t=1}^{T}\Big[\Big(w_1\, k_{1,\beta}^{p} + w_2 k_{1,\beta}^{m} + w_3 k_{1,\beta}^{o}\Big)\ H_t + \Big(w_1\, m_{1,\beta}^{p} + mk_{1,\beta}^{m} + w_3 m_{1,\beta}^{o}\Big) F_t\Big)\Big] \quad (8)$$

Constraints: Constraints on carrying inventory:

$$I_{n(t-1)} - B_{n(t-1)} + Q_{nt} + O_{nt} + S_{nt} - I_{nt} + B_{nt} = \Big(\mathrm{w}_1 D_{nt,\beta}^{p} + \mathrm{w}_2 D_{nt,\beta}^{m} + \mathrm{w}_3 D_{nt,\beta}^{o}\Big)\ for\ \forall n, \forall t \quad (9)$$

Constraints on Labor levels:

$$\sum_{n=1}^{N} i_{n(t-1)}\left(Q_{n(t-1)} + O_{n(t-1)}\right) + H_t - F_t - \sum_{n=1}^{N} i_{nt}\left(Q_{nt} + O_{nt}\right) = 0 \quad for\ \forall t \quad (10)$$

$$\sum_{n=1}^{N} i_{nt}\left(Q_{nt} + O_{nt}\right) \le \left(\mathrm{w}_1 W_{t\,max,\beta}^{p} + \mathrm{w}_2 W_{t\,max,\beta}^{m} + \mathrm{w}_3 W_{t\,max,\beta}^{o}\right) for\ \forall t \quad (11)$$

Constraints on Machine capacity:

$$\sum_{n=1}^{N}(\mathrm{w}_1 r^{p}_{nt\,max,\beta} + \mathrm{w}_2 r^{m}_{nt\,\,max,\beta} + \mathrm{w}_3 r^{o}_{nt\,max,\beta})(O_{nt} + Q_{nt}) \le (\mathrm{w}_1 M^{p}_{t\,max,\beta} + \mathrm{w}_2 M^{m}_{t\,max,\beta} + \mathrm{w}_3 M^{o}_{t\,max,\beta})\ for\ \forall t\ \ for\ \forall \qquad (12)$$

Constraints on Warehouse space:

$$\sum_{n=1}^{N} V_{nt} I_{nt} \le V_{tmax} \quad for\ \forall t \qquad (13)$$

$$Q_{nt}, O_{nt}, S_{nt}, I_{nt}, B_{nt}, H_t, F_t \ge 0\ \ for\ \forall n, \forall t \qquad (14)$$

3.3.3 Improved Mathematical Model for Case 5

Model Assumptions:

i. The values of most of the parameters are uncertain over the next T planning horizon and follow a specific range for each period and simultaneously a random value will be generated from this specific range and it will vary from period to period.
ii. Actual labor levels, machine capacity and warehouse space in each period cannot exceed their respective maximum levels.This capacity levels also follow a specific range.
iii. The forecasted demand over a particular period is also uncertain that will be taken from generating random number from a predefined specific range and it will vary from period to period and it can be either satisfied or backordered, but the backorder must be fulfilled in the next period.
iv. For Every iteration random values will be generated for each of the parameter for evaluates the objective function as well as constraints function and total number of iteration will be evaluated one lac (i=1:100000)

The following notation is used after reviewing the literature and considering practical situations (Wang and Liang, 2004; Masud and Hwang, 1980; Wang and Fang, 2001)

RNG Random number generation

N: Types of products
Z: Total costs ($)
$\widetilde{D}_{nt,i}$: Forecasted demand for nth product in period t at i^{th} iteration of RNG (units)
$\tilde{a}_{nt,i}$: Regular time production cost per unit for nth product in period t at i^{th} iteration of RNG ($ /unit)
Q_{nt}, i: Regular time production volume for nth product in period t at i^{th} iteration of RNG (units)
$\tilde{b}_{nt}, i$: Overtime production cost per unit for nth product in period t at i^{th} iteration of RNG ($ /unit)
O_{nt}, i: Overtime production volume for nth product in period t at i^{th} iteration of RNG (units)
$\tilde{c}_{nt}$: Subcontracting cost per unit of nth product in period t at i^{th} iteration of RNG ($ /unit)
$S_{nt,i}$: Subcontracting volume for nth product in period t at i^{th} iteration of RNG (units)

$\tilde{d}_{nt,i}$: Inventory carrying cost per unit of nth product in period t at i^th^ iteration of RNG ($ /unit)
$I_{nt,i}$: Inventory level in period t for nth product at i^th^ iteration of RNG (units)
$\tilde{e}_{nt}$: Backorder cost per unit of nth product in period t at i^th^ iteration of RNG ($ /unit)
$B_{nt,i}$: Backorder level for nth product in period t at i^th^ iteration of RNG (unit)
$\widetilde{K}_{t,i}$: Cost to hire one worker in period t at i^th^ iteration of RNG ($/man-hour)
$H_{t,i}$: Worker hired in period t at i^th^ iteration of RNG (man-hour)
$\widetilde{m}_{t,i}$: Cost to layoff one worker in period t at i^th^ iteration of RNG ($/man-hour)
$F_{t,i}$: Workers laid off in period t at i^th^ iteration of RNG (man-hour)
i_{nt}: Hours of labor per unit of nth product in period t (man-hour/unit)
$\tilde{r}_{nt,i}$: Hours of machine usage per unit of nth product in period t at i^th^ iteration of RNG
V_{nt}: Warehouse spaces per unit of nth product in period t (ft^2/unit)
$\widetilde{W}_{tmax,i}$: Maximum labor level available in period t at i^th^ iteration of RNG (man-hour)
$\widetilde{M}_{tmax,i}$: Maximum machine capacity available in period t at i^th^ iteration of RNG (machine-hour)
V_{tmax}: Maximum warehouse space available in period t (ft^2)

The related cost coefficients in the objective function frequently are imprecise in nature because some information is incomplete or unobtainable in a medium time horizon. These cost coefficients will be considered randomly by generating random number from predefined particular range of each parameter so accordingly; improved objective function of the proposed model is as follows:

$$\boldsymbol{Min\ Z} = \sum_{n=1}^{N}\sum_{t=1}^{T}\left[\tilde{a}_{nt,i}\,Q_{nt,i} + \tilde{b}_{nt,i}\,O_{nt,i} + \tilde{c}_{nt,i}S_{nt,i} + \tilde{d}_{nt,i}I_{nt,i} + \tilde{e}_{nt,i}B_{nt,i}\right] + \sum_{t=1}^{T}\left(\widetilde{K}_{t,i}H_{t,i} + \widetilde{m}_{t,i}F_{t,i}\right) \tag{15}$$

Constraints: Constraints on carrying inventory:

$$I_{n(t-1)} - B_{n(t-1)} + Q_{nt,i} + O_{nt,i} + S_{nt,i} - I_{nt,i} + B_{nt,i} = \breve{D}_{nt,i} \quad for\ \forall n, \forall t \tag{16}$$

Constraints on Labor levels:

$$\sum_{n=1}^{N} i_{n(t-1)}\left(Q_{n(t-1)} + O_{n(t-1)}\right) + H_{t,i} - F_{t,i} - \sum_{n=1}^{N} i_{nt}\left(Q_{nt,i} + O_{nt,i}\right) = 0 \quad for\ \forall t \tag{17}$$

$$\sum_{n=1}^{N} i_{nt}\left(Q_{nt,i} + O_{nt,i}\right) \le \widetilde{W}_{t\,max,i} \quad for\ \forall t \tag{18}$$

Constraints on Machine capacity:

$$\sum_{n=1}^{N} \hat{r}_{nt,i}\left(O_{nt} + Q_{nt}\right) \le \widehat{M}_{t\,max,i} \ \ for\ \forall t \tag{19}$$

Constraints on Warehouse space:

$$\sum_{n=1}^{N} V_{nt}I_{nt} \le V_{tmax} \quad for\ \forall t \tag{20}$$

$$Q_{nt}, O_{nt}, S_{nt}, I_{nt}, B_{nt}, H_t, F_t \ge 0 \ \ for\ \forall n, \forall t \tag{21}$$

4. Data Collection and Solving Procedures

4.1 Data Collection

Secondary data have been used to demonstrate the practicality of the proposed methodology (Wang and Liang, 2004b). The planning horizon is 4 months long, including May, June, July, and August. The model includes two types of standard ball screw, namely the external recirculation type (product 1) and the internal recirculation type (product 2). According to the preliminary environmental information, Tables 1–6 summarize the forecast demand, related operating cost, and capacity data used in the model. The forecast demand, relevant operating cost, and maximum labor and machine capacity are imprecise numbers with triangular possibility distributions from period to period. Other relevant data are as follows:

1. Initial inventory in period 1 is 400 units of product 1 and 200 units of product. End inventory in period 4 is 300 units of product 1and 200 units of product 2.

2. Initial labor level is 300 man-hours. The costs associated with hiring and layoff are ($8, $10, $11) and ($2.0, $2.5, $3.2) per worker per hour, respectively.

3. Hours of labor per unit for any periods are fixed to 0.05 man-hours for product 1 and 0.07 man hours for product 2. Hours of machine usage per unit for each of the four planning periods are (0.09, 0.10, 0.11) machine-hours for product 1 and (0.07, 0.08, 0.09) machine-hours for product 2. Warehouse spaces required per unit are 2 square feet for product 1 and 3 square feet for product 2.

Table 1: Forecast Demand Data (Existing Table)

Item	Period			
	1	2	3	4
$\widetilde{D}_{1t}$	(900,1000,1080)	(2750, 3000, 3200)	(4600, 5000, 5300)	(1850, 2000, 2100)
$\widetilde{D}_{2t}$	(900,1000,1080)	(450, 500, 540)	(2750, 3000, 3200)	(2300, 2500, 2650)

Table 2: Related Operating Cost Data (Existing Table)

Product	$\tilde{a}_{nt}$($/unit)	$\tilde{b}_{nt}$($/unit)	$\tilde{c}_{nt}$($/unit)	$\tilde{d}_{nt}$($/unit)	$\tilde{e}_{nt}$($/unit)
1	(17, 20, 22)	(26, 30, 33)	(22, 25, 27)	(0.27, 0.30, 0.32)	(35, 40, 44)
2	(8, 10, 11)	(12, 15, 17)	(10, 12, 13)	(0.13, 0.15, 0.16)	(16, 20, 23)

Table 3: Maximum Labor, Machine & Warehouse Capacity Data (Existing Table)

Period	$\widetilde{W}_{t\,max}$(man-hours)	$\widetilde{M}_{t\,max}$(machine-hours)	$V_{t\,max}$(ft^2)
1	(175, 300, 320)	(360, 400, 430)	10,000
2	(175, 300, 320)	(450, 500, 540)	10,000
3	(175, 300, 320)	(540, 600, 650)	10,000
4	(175, 300, 320)	(450, 500, 540)	10,000

Table 4: Weighted Average Data of Forecasting Demand

Item	Period			
	1	2	3	4
$\widetilde{D}_{1t}$	(900*0.17+1000*0.66+1080*0.17) = 966.67	(2750*0.17+ 3000*0.66+ 3200*0.17) = 2991.67	(4600*0.17+ 5000*0.66+ 5300*0.17) = 4983.33	(1850*0.17+ 2000*0.66+ 2100*0.17) = 1991.67
$\widetilde{D}_{2t}$	(900*0.17+1000*0.66+1080*0.17) = 966.67	(450*0.17+ 500*0.66+ 540*0.17) = 498.33	(2750*0.17+ 3000*0.66+ 3200*0.17) = 2991.67	(2300*0.17+ 2500*0.66+ 2650*0.17) = 2491.67

Table 5: Related Operating Costs from Weighted Average Method (Considered)

Product	$\tilde{a}_{nt}$($/unit)	$\tilde{b}_{nt}$($/unit)	$\tilde{c}_{nt}$($/unit)	$\tilde{d}_{nt}$($/unit)	$\tilde{e}_{nt}$($/unit)
1	(17*0.17+ 20*0.66+ 22*0.17) =19.83	(26*0.17+ 30*0.66+ 33*0.17) = 29.83	(22*0.17+ 25*0.66+ 27*0.17) = 24.83	(0.27*0.17+ 0.30*0.66+ 0.32*0.17) = 0.30	(35*0.17+ 40*0.66+ 44*0.17) = 39.83
2	(8*0.17+ 10*0.66+ 11*0.17) = 9.83	(12*0.17+ 15*0.66+ 17*0.17) = 14.83	(10*0.17+ 12*0.66+ 13*0.17) = 11.83	(0.13*0.17+ 0.15*0.66+ 0.16*0.17) = 0.15	(16*0.17+ 20*0.66+ 23*0.17) = 19.83

Table 6: Conversion of Three Values into Single Value (Considered)

Period	$\widetilde{W}_{t\,max}$(man-hours)	$\widetilde{M}_{t\,max}$(machine-hours)	$V_{t\,max}$(ft^2)
1	(175*0.17+ 300*0.66+ 320*0.17) = 282.50	(360*0.17+ 400*0.66+ 430*0.17) = 398.33	10,000
2	(175*0.17+ 300*0.66+ 320*0.17) = 282.50	(450*0.17+ 500*0.66+ 540*0.17) = 498.33	10,000
3	(175*0.17+ 300*0.66+ 320*0.17) = 282.50	(540*0.17+ 600*0.66+ 650*0.17) = 598.33	10,000
4	(175*0.17+ 300*0.66+ 320*0.17) = 282.50	(450*0.17+ 500*0.66+ 540*0.17) = 498.33	10,000

For the implementation of **proposed approach** data have been modified as follows in table 7- 9 are given below.

Table 7: Forecasted Demand Range (Units)

Item	Period			
	1	2	3	4
$\widetilde{D}_{1t}$	(900 - 1080)	(2750 - 3200)	(4600 - 5300)	(1850 - 2100)
$\widetilde{D}_{2t}$	(900 - 1080)	(450 - 540)	(2750 - 3200)	(2300 - 2650)

Table 8: Related Operating Cost Range

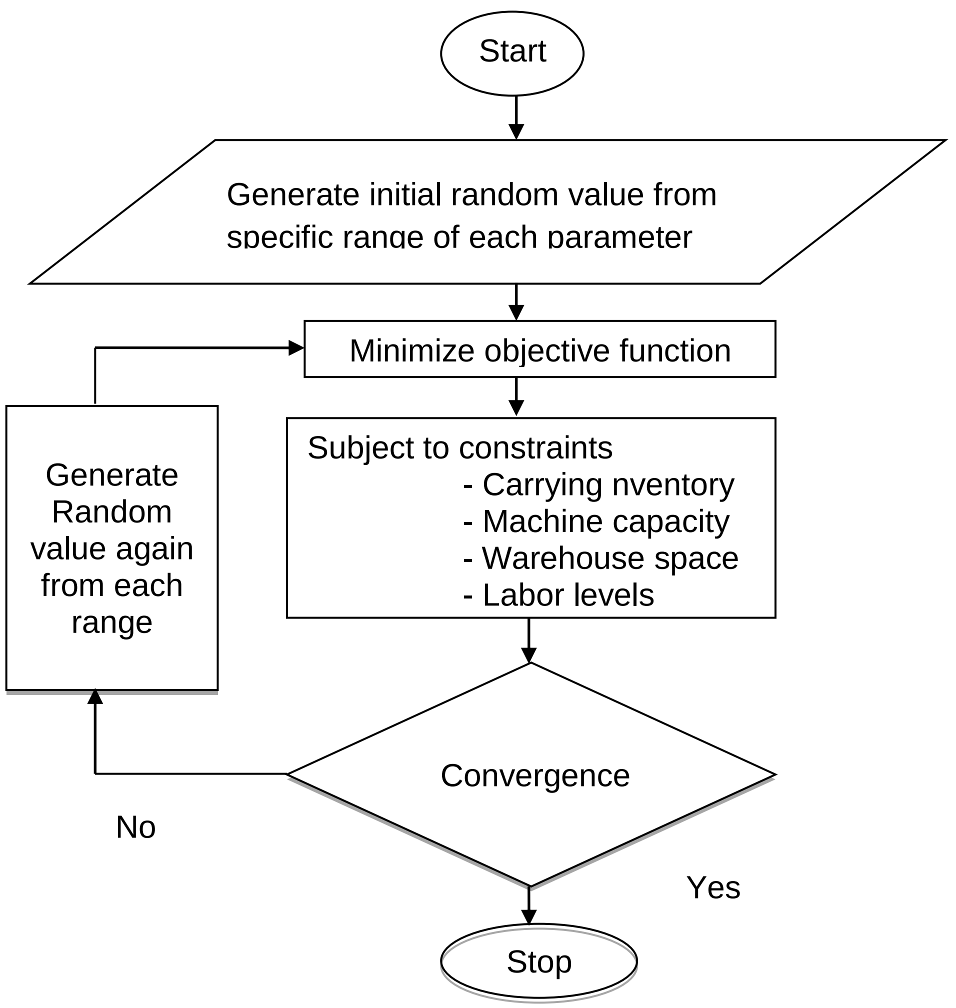
Product	$\tilde{a}_{nt}$($/unit)	$\tilde{b}_{nt}$($/unit)	$\tilde{c}_{nt}$($/unit)	$\tilde{d}_{nt}$($/unit)	$\tilde{e}_{nt}$($/unit)
1	(17 - 22)	(26 - 33)	(22 - 27)	(0.27 - 0.32)	(35 - 44)
2	(8 - 11)	(12 - 17)	(10 - 13)	(0.13 - 0.16)	(16 - 23)

Table 9: Maximum Labor, Machine, and Warehouse Capacity Range

Period	$\widetilde{W}_{t\,max}$(man-hours)	$\widetilde{M}_{t\,max}$(machine-hours)	$V_{t\,max}$(ft^2)
1	(175 - 320)	(360 - 430)	10,000
2	(175 - 320)	(450 - 540)	10,000
3	(175 - 320)	(540 - 650)	10,000
4	(175 - 320)	(450 - 540)	10,000

4.2 Solving Procedures

Step 1: Formulate the LP model for the APP decision problem according to Eqs. (1) to (7) for four periods for two products. *Step 2:* Use the pessimistic, most likely and optimistic values from table (1 - 3) in current model one by one and determine the minimum cost. *Step 3:* Write and run the program according to step 2 in matlab computer software version 2010 and get the optimal solution. *Step 4:* Convert the three values into one precise value shown in table 4-6 by using weighted average method and then use it like as step 2 in order to get the result for case 4. *Step 5:* (Proposed approach): Generate random number using predefined range shown in modified data in table 7–9 for the corresponding APP parameters and accordingly write and run the program using for loop (for 100,000 iterations) in matlab computer software and get the optimal result. Outline of the proposed approach is mentioned as follows:



5. Results and Findings

5.1 Results

It is mentioned that fuzzy triangular membership function is applied in previous research work and acoordingly three objective values are obtained. Now current obtained results considering five cases are compared with previous existing results separately as follows in table **10-13.**

Table 10: Comparison of Results with Previous Existing Result 1

(I) Type of Value	(II) Obtained Result ($)	(III) Previous Result 1($)	Difference between [(II)-(III)]	Change % Compared to (III)
Pessimistic value	449430	276839	172591	62.34%
Most likely value	289890	276839	13051	4.71%
Optimistic value	258130	276839	-18709	**(-) 6.76%**
Combinely value	452960	276839	176121	63.62%
Random value	238190	276839	-38649	**(-) 13.96%**

From table 10, it is found that if previous existing result 1 is compared with the current obtained result based on considering pessimistic value, most likely value and weighted average value then it is noticed that 62.34%, 4.71% and 63.62% cost is increased than existing result 1 correspondingly. Beside it is observed that if optimistic data are considered then 6.76% cost is decreased than existing result 1. On the other hand it it remarkable that if random value (Proposed approach) is considered then 13.96% cost is decreased than existing result 1. Therefore it may be said that case no. 5 is always better in terms of reducing total aggregate production cost compare with other four cases.

Table 11: Comparison of Results with Previous Existing Result 2

(I) Type of Value	(II) Obtained Result ($)	(III) Previous Result 2 ($)	Difference between [(II)-(III)]	Change % Compared to (III)
Pessimistic value	449430	326499	122931	37.65%
Most likely value	289890	326499	-36609	**(-)11.21%**
Optimistic value	258130	326499	-68369	**(-)20.94%**
Combinely value	452960	326499	126461	38.73%
Random value	238190	326499	-88309	**(-)27.05%**

From table 11, it is found that if previous existing result 2 is compared with the current obtained result based on considering pessimistic value, and weighted average value (combine value) then it is noticed that 37.65% and 38.73% cost is increased than existing result 2 correspondingly. Beside it is observed that if most likely, optimistic data and random values are considered then 11.21%, 20.94% and 27.05% cost is decreased correspondingly than existing result 1. Throughout this observation it is remarkable that if random value (Proposed approach) is considered then 27.05% cost is decreased than existing result 2 which is the maximum cost reducing rate among these 5 consdierations. Therefore it may be said that case no. 5 (considering random number generation from a specific limit) is always better in terms of reducing total aggregate production cost compare with other four cases.

Table 12: Comparison of Results with Previous Existing Result 3

(I) Type of Value	(II) Obtained Result ($)	(III) Previous Result 3 ($)	Difference between [(II)-(III)]	Change % Compared to (III)
Pessimistic value	449430	356339	93091	26.12%
Most likely value	289890	356339	-66449	(-)18.65%
Optimistic value	258130	356339	-98209	(-)27.56%
Combinely value	452960	356339	96621	27.11%
Random value	238190	356339	-118149	(-)33.16%

From table 12, it is found that if previous existing result 3 is compared with the current obtained result based on considering pessimistic value and weighted average value (combine value) then it is noticed that 26.12% and 27.11% cost is increased than existing result 3 correspondingly. Beside it is observed that if most likely, optimistic and random values are considered then 18.65%, 27.56% and 33.16% cost is decreased correspondingly than existing result 3. Throughout this observation it is remarkable that if random value (Proposed approach) is considered then 33.16% cost is decreased than existing result 3 which is the maximum cost reducing rate among these 5 consdierations. Therefore it may be said that case no. 5 (considering random number generation from a specific limit) is always better in terms of reducing total aggregate production cost compare with other four cases.

Table 13: Comparison of Results between Case 1, 2, 3 & 4 with Case 5

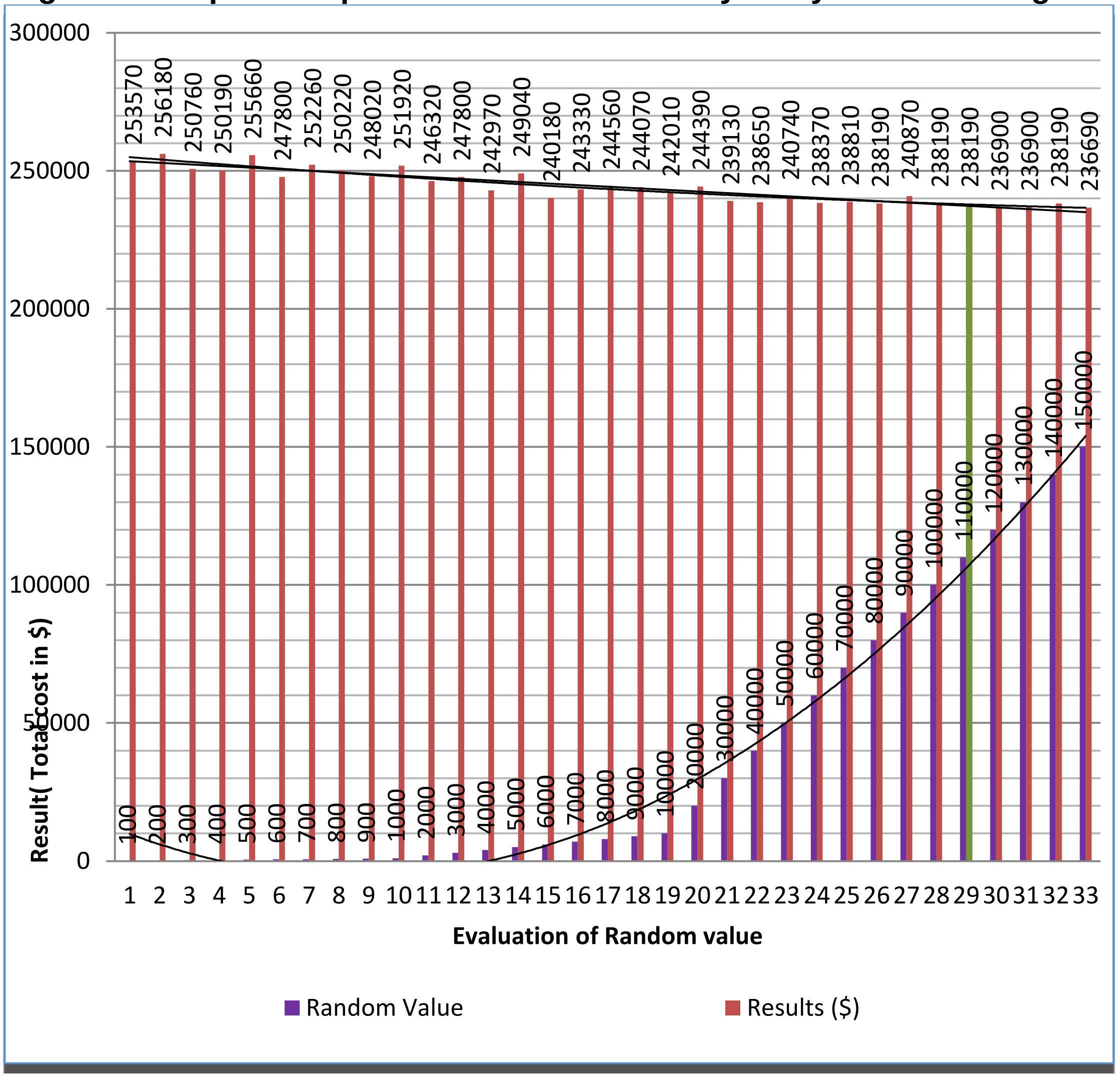
[I] Case No.	[II] Obtained result ($)	[III] Obtained results ($) from **Case 5**	Differences($) [II-III]	Change % compared to [II]
Case 1	449430	238190	211240	47.00%
Case 2	289890	238190	51700	17.83%
Case 3	258130	238190	19940	7.72%
Case 4	452960	238190	214770	47.41%

From table 13, it is found that based on proposed approach that's mean Case no. 5 produced always lowest cost compare to other four cases. Therefore it is said that Case no. 5 is always cost effective approach compare to Case no.1, Case no. 2, Case no. 3 and Case no. 4 respectively. So it can be said that case 5 generate different result which is relatively better from any previous approach. It gives a unique idea and advance knowledge for practioners as well as researchers.

Sensitivity Analysis

It is noted that in this research work random values are considered that have been generated randomly from a predefined specific range for evaluating the proposed approach. There is infinite no. of random values in each of the range. So there is question how much values should be evaluated. For searching this answer consequtively from 1 to 150000 random values are evaluated. It is noticed in the early period that is when evaluating small amount of random values its produce relatively worse result than others higher number. To clarify this notion evaluation of random number and their corresponding results are represented as follows in figure 1.

Figure 1: Graphical representation of sensitivity analysis in Bar Diagram



5.2 Findings

Throughout the analysis and observation it is seen that proposed approach Case no. 5 (Random no. generation from a specific limit) is always producing better result compare to other four approaches of Case 1, Case 2, Case 3 and Case 4. It is found that applying Case 5 total Aggregate Production Planning cost around 47.00%, 17. 83%, 7.72% and 47.41% is decreased compare to Case 1, Case 2, Case 3 and Case 4 respectivly. Besides if result based on Case 5 is compared to previously obtained results it is found that around 13.96%, 27.05% and 33.16% of total aggregate production planning cost is decreased than previous results 1, 2 and 3 respectively. To sums up it is said that proposed approach Case no. 5 is always responsible for producing better result compare to others approaches. Hence, the authors want to conclude that proposed approach not only cost effective approach but also a novel idea to solve the aggregate production planning problem under considering uncertain environment.

6. Conclusion

In this research work the proposed approach draws the attention of the APP practioner how to uncertain value is considered for getting APP decision. It focused advance knowledge for the researchers as well as practioner. Presently most of the researchers as well as practioner used certain value for getting APP decision regarding on uncertain environment. But by this proposed approach APP practioners can be used their uncertain value in their uncertain environment directly. Iteratively random value is used randomly for evaluating the APP model until the satisfactory result is found out. As it is done by matlab computer software so a lot of random value can be evaluated within very short time. Hence the result by this approach may be more acceptable to practioners and to researchers. Finally it may be concluded that this new approach may be novel idea and open a new dimension for the researchers as well as practioners in future for handling uncertain environment for other field also.

7. Recommendations for Further Research

Research is continuous and never ending process. Researchers always try to find out a unique result overcoming past limitations. But still there remains scope to carry out the research work on the same area for improving the result. In this research work author has been tried to overcome some limitations of past research work to get better APP result. The major limitation of this research work is that for justifying proposed approach secondary data have been used. Besides, only Linear Programming (LP) is applied to evaluate the proposed approach but there are various types of algorithm such as genetic algorithm, particle swarm algorithm, ant colony optimization that may be used to evaluate the proposed approach for getting better result.

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