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Business Investment and Economic Depreciation

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In this paper, we show that economic depreciation decreases value maximizing corporate hurdle rates to encourage both maintenance and growth investments. Economic depreciation without maintenance decreases the upside potential of future growth opportunities upon stochastically improved profitability. Deferred maintenance undertaken upon improved profitability requires scarce corporate resources at the expense of business expansion investments. Managers forestall this loss of growth option value with maintenance investments even with modest current business returns. While economic depreciation encourages both maintenance and growth for existing businesses, it impedes the start of new ventures in the first instance.

Field of Research: Economic depreciation; Corporate investment; New venture start

JEL Classification: D81, G31, O16

1. Introduction

In this paper, we show that economic depreciation decreases value maximizing corporate hurdle rates to encourage both maintenance and growth investments. Economic depreciation without maintenance decreases the upside potential of future growth opportunities upon stochastically improved profitability. Deferred maintenance undertaken upon improved profitability requires scarce corporate resources at the expense of business expansion investments. Managers forestall this loss of growth option value with maintenance investments even with modest current business returns. While economic depreciation encourages both maintenance and growth for existing businesses, it impedes the start of new ventures (new venture start) in the first instance.

Hicks (1939) defines income as the per period consumption-potential of an asset while preserving wealth. Economic depreciation is the reduced ability of an asset to generate cash flows (for consumption possibly) due to wear/tear and/or entropy. Maintenance preserves Hicksian income by offsetting economic depreciation. McGrattan and Schmitz (1999) report that

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repair and maintenance equals 6% of Canadian GNP from 1961-1993, 48% of new equipment spending, and 20% of new structure spending. These large amounts identify maintenance and depreciation as important business investment research topics.

The majority of the academic maintenance research appears in the engineering and management science literatures (see Pham and Wang's (1996) survey). Nickell (1975) summarizes the early capital replacement literature. Licandro and Puch (2000) investigate the role of maintenance in business cycles. Since corporations can expense some maintenance and all repairs, McGratten and Smith (1999) argue that maintenance has a tax advantage over new capital spending that makes these outlays substitutes. In Boucekkine and Ruiz-Tamarit (2003), maintenance and new capital spending are complements because economic depreciation increases with capital utilization.

Mauer and Ott (1995) show that depreciation can either increase or decrease optimal equipment replacement times to prevent increasing costs arising from declining equipment reliability. However, decreased replacement times arise only for extreme depreciation rates, and therefore, the essence of their result is that depreciation discourages asset replacement. If tax depreciation (that is, the amount of depreciation that is deductible for income tax purposes) exceeds economic depreciation, a government tax-recapture upon asset salvage delays equipment replacement and, thus, depreciation delays investment in the first instance.

Dixit and Pindyck (1994, pp. 199-205) model depreciation as a Poisson death process. Unexpected equipment failure reduces the period over which an investment recoups cost, which increases the profit threshold for a solitary irreversible investment. The possibility of asset replacement moderates the delaying force of irreversibility but the first effect dominates so that economic depreciation implicit in the Poisson death process discourages investment.

In the business investment literature, we are the first to predict a positive relation between business investment and economic depreciation for mature businesses and the first to predict that economic depreciation has opposite impacts for managerial expansion decisions of a mature business and the managerial start decision for a new venture. Mature businesses have existing operations arising from in-place assets, while new ventures have neither existing operations nor in-place assets. Managers of mature businesses maintain depreciable assets and even grow their businesses to prevent the adverse impact of economic depreciation on growth options so that economic depreciation encourages corporate investment. On the contrary, economic depreciation increases the "sunk costs" irreversible investment, which delays the start of a new venture in the first instance. These results have important implication for the decision-making process of business managers.

In Section 2 of this paper, we extend Blazenko and Pavlov's (2009) growth model to incorporate economic depreciation and develop relations between profit, capital, economic depreciation, maintenance, growth, and investment return. Section 3 derives value maximizing hurdle rates for maintenance and growth. We show that economic depreciation promotes corporate investment when maintenance-neglect erodes growth option value. While economic depreciation encourages maintenance and expansion investments for mature businesses, it impedes the start of a new venture in the first instance. Section 4 summarizes, concludes, and proposes empirical tests for future investigation.

2. Economic Depreciation

2.1 Operating Profit, Maintenance, and Growth

Because both maintenance and growth investments are ancillary to an existing business (so that existing business profitability encourages both), we model growth and maintenance as complementary investments. Bierman and Smidt (1992) describe a procedure to measure economic depreciation consistent with discounted cash flow valuation. If forecast cash flows decrease at a constant rate without maintenance, then, declining balance depreciation at the same rate represents economic depreciation. We use this depreciation modeling in this paper.

Consider a business that currently earns operating profit[1] at the per annum rate, $X_t > 0$. The operating profit process is,

$$\frac{dX}{X} = \begin{cases} -\rho dt + \sigma dz, & \text{no maintenance or growth} \\ \\ gdt + \sigma dz, & \text{maintenance and growth} \end{cases} \tag{1}$$

Where $\rho \geq 0$ is economic depreciation, $g \geq 0$ is the profit growth rate when the manager grows the business, σ is operating profit volatility, and *dz* is a Gauss-Weiner stochastic increment. On the upper branch of Equation (1), the manager neglects maintenance (and growth) and profit declines at the rate of economic depreciation, *ρ*. On the other hand, the decline parameter, *ρ*, does not appear on the lower branch because maintenance offsets economic depreciation and instead, growth investments grow profit at the per annum rate, *g*. A common branch to describe maintenance and growth presumes that if maintenance is value creating then growth investment is value creating as well, and vice versa. The manager endogenously maintains and grows the business. Profit maintenance and growth is neither immediate, spontaneous, nor without cost, but requires capital investment that we describe next.

2.2 Capital, Maintenance, and Growth

Incremental investment brings two benefits to a business: it prevents economic depreciation and it grows a business. Investment provides benefits indefinitely and, thus, firms capitalize this investment expenditure for accounting purposes.[2] Let *B* represent capitalized investment expenditures net of depreciation. If the manager maintains and grows the business, then capital stock, *B*, grows. Otherwise, capital stock declines at the economic depreciation rate, *ρ*, without maintenance. Capital stock, *B*, also measures replacement cost, which increases with asset quality (the ability to generate future profit). We restrict incremental investment to a fraction of existing capital, $\rho + g$. Incremental investment, $\rho B dt$, over the upcoming instant, *dt*, prevents economic depreciation and growth expenditure, $gBdt$, grows the business at the per annum rate *g*. The investment constraint, $\rho + g$, ensures that incremental investment is proportional to firm size. Briggs (1992) investigates constrained maintenance in the building industry.

We presume that investments are irreversible.[3] Poor corporate profitability is particularly onerous for firms with irreversible investments that are subject to economic depreciation.

Illiquidity prevents managers from disposing under-performing assets and, in addition, depreciation worsens corporate profitability without maintenance.

At the expense of some additional notation, we could model *depreciation-in-use* that requires a higher depreciation rate for more intensely used assets. For example, we might use a higher depreciation rate when the firm grows and a lower rate otherwise. Since this modeling does not change the substance of our results, we consider only a single economic depreciation rate, *ρ*.

Capital stock, *B*, increases when the manager maintains and grows the business and declines otherwise. The capital stock process is,

$$\frac{dB}{B} = \begin{cases} -\rho dt, & \text{no maintenance or growth} \\ \\ gdt, & \text{maintenance and growth} \end{cases} \qquad (2)$$

Economic depreciation on the upper branch of Equation (2) represents the decline in the quality of the asset as its profit-generating ability deteriorates without maintenance. An impact of this decline is that when the manager eventually maintains and grows the business once more, the cost of incremental investment, $(\rho + g)Bdt$, is also lower due to the incremental investment proportionality constraint. As we show in the upcoming subsection, the lower profitability and the lower replacement cost mean that replacement/growth investments do not have returns lesser than in-place assets because of a maintenance-neglect event.

On the lower branch of Equation (2), when the manager maintains and grows the business, capital stock, *B*, grows at the per annum rate *g*. Incremental investment, $(\rho + g)Bdt$, makes capital stock grow by the dollar amount, $gBdt$. On the other hand, maintenance investment, ρBdt, prevents economic depreciation but it does not increase capital stock. Equivalently, capital stock, *B*, declines by ρBdt due to the economic depreciation of the in-place asset, while it increases by ρBdt because of maintenance investment and, hence, there is no net impact on the capital stock, *B*.

2.3 Investment Return

Maintenance and expansion investments are scaled versions of in-place assets. A consequence of this scaling is that the rate of return to ancillary investments (maintenance and growth) equals that of in-place assets. Net-return, *Y* – ρ, is gross return on capital, $Y \equiv \frac{X}{B}$, less economic depreciation, *ρ*. Ignoring subsequent growth or maintenance investments, this expression means that in-place assets generate indefinite operating profit that declines at the economic depreciation rate, *ρ*. Since we make no distinction between capital stock that originates from business start or from subsequent growth and maintenance investments, net-return for these investments also equals this expression. To see this equality, note that if ΔB is a maintenance or growth investment, then the rate of return (the IRR) satisfies, $\frac{Y\Delta B}{IRR+\rho} - \Delta B = 0$. Rearrange this equation to find $IRR = Y - \rho$, which means that maintenance and growth investments inherit gross return, Y, from in-place assets.

Economic depreciation is a characteristic of an individual investment. Without maintenance, which is a separate investment, operating profit declines at the rate of economic depreciation, *ρ*. Since individual investments have this feature, economic depreciation, *ρ*, appears as a net-return component, $IRR = Y - \rho$. On the other hand, growth, *g*, is not an individual investment feature; rather, it is an corporate characteristic arising from ongoing managerial capital investment. As a result, the growth parameter, *g*, does not appear in the incremental investment return, $IRR = Y - \rho$. A solitary investment has no growth, *g*, on its own. Rather, businesses growth because they make incremental growth investments.

If gross return, Y, were to grow or decline over time, there would be an incentive for managers to accelerate or delay maintenance and growth investments. However, gross return neither grows nor declines: ROC is a martingale, $E[\tilde{Y}_t] = Y_0$. To verify this assertion, use Ito's lemma and the branches of Equations (1) and (2),

$$\mathrm{d}\tilde{Y} = \frac{\partial Y}{\partial X}dX + \frac{\partial Y}{\partial B}dB = \begin{cases} -\rho Y dt + Y\sigma dz + \rho Y dt = Y\sigma dz, & \textit{no maintenance or growth} \\ -gYdt + Y\sigma dz + gYdt = Y\sigma dz, & \textit{maintenance and growth} \end{cases} \tag{3}$$

Given that the upper and lower branches of Equation (3) equal one another, and there is no "drift", ROC is a lognormal martingale. The fact that ROC neither grows nor declines means that gross return, *Y*, does not change during either periods with maintenance or periods without maintenance or growth other than due to random profitability increments, σdz. Also, neither the economic depreciation rate, *ρ*, nor the growth factor, *g*, alters the distribution of future gross return, $\tilde{Y}_t$, because neither parameter appears in Equation (3). The reason for this result is that both the numerator and the denominator of gross return, $Y = {}^{X}\!/_{B}$, change at the same rate during periods with or without either maintenance or growht (-*ρ* and g, respectively). Neither maintenance/growth nor maintenance-neglect influences the rate of return for subsequent maintenance and growth investments. The maintenance or growth history of in-place assets has no influence on the rate of return to newly acquired assets from outside the firm.

2.4 Assumptions

Managers finance maintenance and expansion investments with equity. We do not consider financing costs, taxes, or other frictions except to the extent that they are encompassed within the investment proportionality constraint, $\rho + g$. There is a vast real-options literature on transactions cost impediments to business investment. One can add alternative and/or additional corporate features in an extended version of our model. The advantage of our simple investment representation is that it allows a closed form solution for the value maximizing investment hurdle rate. At the same time, our model is not stylized, and closely represents the decision environment of practicing managers.

3. Value Maximizing Hurdle Rates for Maintenance and Expansion Investment

We use the valuation methodology of Goldstein et al. (2001) to value a business that endogenously maintains in-place assets and grows. Goldstein, Ju, and Leland presume spontaneous profit growth without capital investment, and therefore, they consider neither maintenance nor capital growth. Because our analysis extends Blazenko and Pavlov's (2009) business growth model incorporating economic depreciation and maintenance, our continued description of it is brief and is contained primarily in Appendix A. Instead, we focus on results novel to economic depreciation and maintenance investment.

3.1 The Value Maximizing Maintenance and Growth Hurdle Rate

The value maximizing net-return hurdle rate for maintenance and growth from Appendix A is,

$$\xi^* - \rho = (r^* + \rho)\left[\frac{r^* - g}{r - g}\right]\left[\frac{\alpha}{\alpha - 1}\right]\left[\frac{\lambda}{\lambda - 1}\right] - \rho \qquad (4)$$

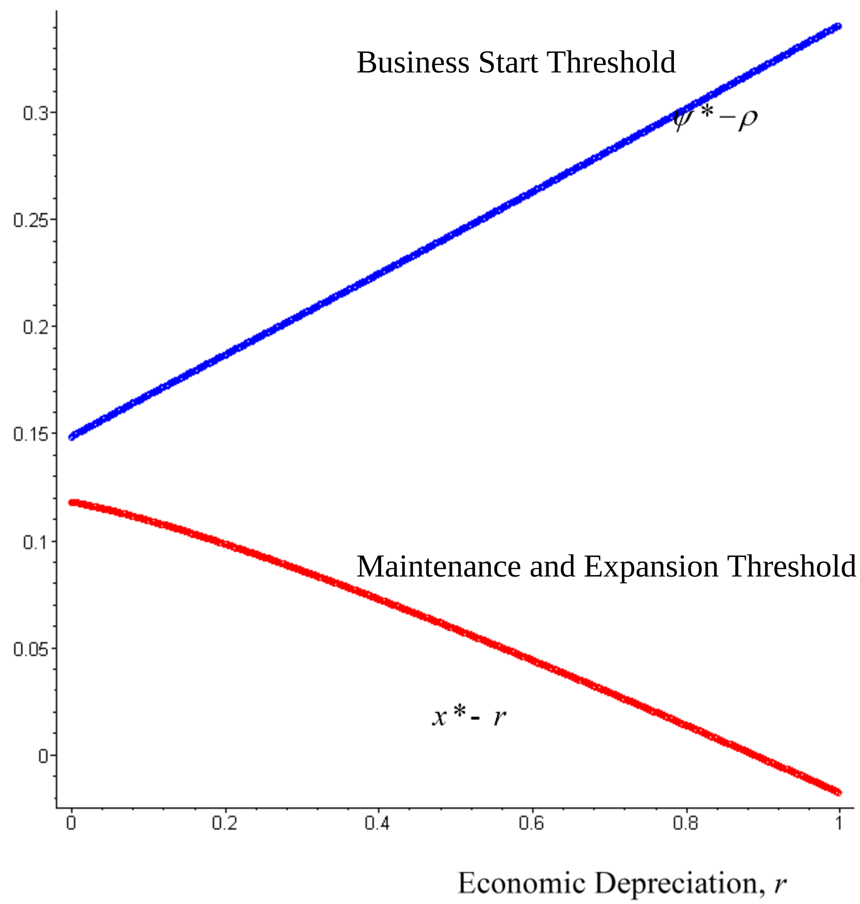
Where, $r^* \equiv r + \theta\sigma_{xc}$, *r* is the riskless interest rate in the economy, $\theta \geq 0$ is the coefficient of constant relative risk aversion for a representative investor[4] who establishes the price of risky financial assets in the economy, σ_{xc} is the covariance of the log of operating profit, *X*, with the log of aggregate consumption in the economy, which follows a geometric Brownian motion. Equation (A4) in Appendix A gives definitions for the parameters α and λ. The expression, $r^* \equiv r + \theta\sigma_{xc}$, is the expected return or the cost of capital for a hypothetical non-growing firm with zero economic depreciation, $\rho = 0$. In this case, without maintenance and growth-leverage, expected return is invariant to return, *Y*. Since investors own equities in diversified portfolios, they focus on incremental portfolio risk for expected return. This incremental risk is measured by covariance; hence, the risk premium, $\theta\sigma_{xc}$, depends on covariance, σ_{xc}. See Brealey et al. (2006, chapter 9) for a discussion of the impact of covariance on business operating risk. For expositional purposes, we presume non-negative covariance risk, $\sigma_{xc} \geq 0$. In this case, expected return for the non-growing firm with zero economic depreciation exceeds the riskless rate, $r^* \geq r$. Mathematical convergence for the business valuation Equation (A3) in appendix A requires that the riskless interest rate exceeds the growth factor,[5] $r > g$.

The manager maintains assets and grows the business when net-return, $Y - \rho$, exceeds the value maximizing net threshold, $\xi^* - \rho$, in Equation (4), or equivalently, when gross return exceeds the gross return threshold, $Y \geq \xi^*$. Otherwise, the manager suspends growth and maintenance and wait for net-return to exceed the threshold. Setting $\rho = 0$ in Equation (4) yields Blazenko and Pavlov's (2009) value maximizing business expansion hurdle rate without economic depreciation.

In a dynamic setting, a manager can suspend and restart maintenance and growth investments indefinitely and at any future time. There are two important forces that economic depreciation, *ρ*, has on the value maximizing dynamic net hurdle rate, $\xi^* - \rho$, in Equation (4) for maintenance and growth investments. First, the combination of irreversibility and leverage risk encourages managerial deferral of these investments. However, in a dynamic business environment where a manager can temporarily suspend these investments, the impact of leverage and irreversibility on managerial deferral is greatly reduced. In Equation (4), this reduction is represented by the ratio $\frac{\lambda}{\lambda-1}$, which is less than one. Second, maintenance-neglect destroys growth option value because a manager must offset this maintenance-neglect with constrained maintenance before the firm can benefit from true growth investments that enhance profitability. This adverse impact of maintenance-neglect discourages managers from suspending maintenance and growth investments and, other things equal, decreases the value maximizing dynamic net hurdle rate, $\xi^* - \rho$. The second force dominates the first.

In Appendix B, we prove that the maintenance and growth net-return threshold, $\xi^* - \rho$, in Equation (4) decreases with the economic depreciation rate, ρ. Figure 1 illustrates this result with a numerical example. A falling net return hurdle rate with economic depreciation means that marginal maintenance and growth investments only partially cover economic depreciation. For example, if economic depreciation is zero, $\rho = 0$, and other parameter equal g = 0.03, r = 0.05, $\sigma = 0.2$, and, $\theta\sigma_{x,c}$ = 0.07, the value maximizing gross hurdle rate, ξ^*, in Equation (4) is 11.82%. On the other hand, with the same parameter values, but with economic depreciation of 10% per annum, $\rho = 0.10$, the value maximizing gross hurdle rate, ξ^*, is 20.95%. The increase in the gross hurdle rate, 20.95% - 11.82% = 9.13% per annum, is not sufficient to cover economic depreciation of 10% per annum so that the net hurdle rate, $\xi^* - \rho$, declines. Managers who have business assets with great economic depreciation are more willing to accept investments with marginal net return to prevent the destruction of growth option value arising from maintenance-neglect.

Figure 1



Value Maximizing Maintenance and Expansion Net-return Hurdle Rate, $\xi^* - \rho$, and New Business Net-return Start Boundary, $\psi^* - \rho$. Parameters: g = 0.03, r = 0.05, $\theta\sigma_{x,c}$ = 0.07, σ = 0.2.

3.2 New Business Start

McDonald and Siegel (1986) have a classic research article on waiting to invest. However, they consider neither economic depreciation nor its impact on growth option value. Prior to new business start, *X*, is "potential" earnings that a business accrues if, hypothetically, the business were to start immediately. Let *B* be the capital outlay to start the business. Because economic depreciation is a feature of in-place asset use, and the business has yet to acquire any assets, potential profit, *X*, is not subject to economic depreciation. Also, because profit growth requires capital growth, and no such investment has yet been made, profit, *X*, does not grow,

$$\frac{dX}{X} = \sigma dz \qquad (5)$$

Let ψ^* be the value maximizing new business gross return start threshold. When $Y < \psi^*$, the manager awaits new business start. At the first passage of *Y* to ψ^*, the manager starts the new business by making the capital investment, *B*. Appendix C develops the following relation satisfied by the value maximizing new business net return start threshold, $\psi^* - \rho$,

$$\psi^* - \rho = \left[\frac{r^* - g}{r - g}\right]\left[\frac{\omega}{\omega - 1}\right]\left[(r + \rho) + \left(\frac{\rho + g}{1 - \lambda}\right)\left(\frac{\alpha}{\alpha - \lambda}\right)\left(\frac{\lambda}{\alpha} - 1\right)\left(\frac{\psi^*}{\xi^*}\right)^{\lambda}\right] - \rho \qquad (6)$$

Where Equation (A8) in Appendix C defines the parameter ω.

Except in special cases that are not interesting for our purposes, there is no closed form solution for the gross return start threshold, ψ^*, from Equation (6). However, once the parameters r, g, $\theta\sigma_{x,c}$, σ, and ρ have been chosen, the gross return start threshold, ψ^*, is easily calculated. Figure 1 illustrates the net-return start threshold, $\psi^* - \rho$, for g = 0.03, r = 0.05, $\theta\sigma_{x,c}$ = 0.07, σ = 0.2, and for economic depreciation rates, ρ, between 0 and 1.

Economic depreciation adversely impacts in-place assets and the growth opportunities of existing business; however, it does not impact either potential assets or potential growth option value of a new business that has not yet been started. Irreversibility of in-place assets forces a manager to bear the burden of economic depreciation and maintain assets to forestall growth option value deterioration. This need does not exist for assets not yet in-place for a business not yet started, and therefore, the new business return start threshold, $\psi^* - \rho$, exceeds the maintenance and growth threshold, $\xi^* - \rho$. As soon as the business has started, the manager has to maintain and grow the business even if its profitability falls.

The discussion above indicates the managerial imperative for maintenance and growth investments for an operating business with in-place assets is greater than for new business start. Managerial latitude for new business start timing is greater than for subsequent maintenance and growth investments. Results in McDonald and Siegel (1986) indicate that managers defer business-start to avoid downside risk: the possibility that managers become burdened with irreversible investments for which profitability has fallen since business start. In our analysis, three corporate characteristics accentuate downside risk arising from low profitability: investment irreversibility, leverage, and, growth option value depreciation from maintenance-neglect. First, economic depreciation, ρ, increases investment irreversibility. Not only that a manager must invest in these assets to start the business, but these assets must also be replaced over time due to depreciation. Second, maintenance investments that are required to offset economic depreciation increase leverage. Last, economic depreciation devalues not only assets, but also growth opportunities. Consequently, higher economic depreciation, ρ, increases downside side risk and reduces managerial incentive for business start. The increase in the net-return business-start threshold, $\psi^* - \rho$, with economic depreciation, ρ, in Figure 1 illustrates this reduced incentive.

A numerical example illustrates the above discussion. If economic depreciation is zero, $\rho = 0$, and with parameters values we presume above, g = 0.03, r = 0.05, $\sigma = 0.2$, and, $\theta\sigma_{x,c}$ = 0.07, then the value maximizing gross return business-start hurdle rate, ψ^*, in Equation (6) is 14.87%. On the other hand, with the same parameter values, but with economic depreciation of 10% per annum, $\rho = 0.10$, the gross return business-start hurdle rate, ψ^*, increases to 26.81%. The increase in the gross return business-start hurdle rate, 26.81% - 14.87% = 11.94% per annum, exceeds economic depreciation of 10%. This excess amount reflects managerial avoidance of increased downside risk, which delays business start beyond the point that is necessary to cover greater economic depreciation, ρ.

If profitability is sufficiently great, a manager accepts the burden of economic depreciation and starts a new business, $Y - \rho \geq \xi^* - \rho$. Both the growth option value preservation force for an existing business and the investment delaying force for a business not yet started are greater for higher economic depreciation, *ρ*. Figure 1 shows that the difference, $\psi^* - \xi^*$, between the net-return start threshold for a new business, $\psi^* - \rho$, and the net-return maintenance and growth threshold, $\xi^* - \rho$, for an existing business increases with economic depreciation, *ρ*. Diverging return thresholds for new business start and expansion means that, other things equals, a manager both delays new business start longer and maintains and grows an existing business longer with greater economic depreciation, *ρ*.

4. Conclusion

In these concluding comments, we summarize the results in our study and anticipate future research by suggesting ways that these results might be subject to empirical testing. The current investment literature generally suggests that economic depreciation discourages corporate investment. Our paper contributes to and differs from the ezxisting investment literature because we are the first to jointly examine the impact of economic depreciation on business expansion for a mature business and the decision to start a business for new ventures. This paper also differs from the current investment literature by showing that economic depreciation encourages rather than discourages business investment for mature business.

By extending Blazenko and Pavlov's (2009) growth model to incorporate economic depreciation, we show in this paper that economic depreciation decreases the value maximizing net hurdle rate for irreversible corporate maintenance and expansion investment. Economic depreciation encourages both maintenance and business expansion. Unheeded economic depreciation decreases the upside potential of future growth opportunities upon stochastically improved profitability. Managers forestall this decrease with maintenance and expansion even when these investments have modest returns. In the limit, as the economic depreciation rate approaches one, the value maximizing hurdle rate net of depreciation becomes negative. While economic depreciation encourages existing-business ancillary investment – maintenance and expansion – it impedes new business start in the first instance.

Managerial value maximizing investment decisions jointly determine corporate profitability and capital. When return on capital is great, managers maintain and grow their businesses, resulting in an increase in both capital and profit. On the other hand, when return on capital is low, managers neither maintain nor grow; hence, economic depreciation decreases both capital and profit. In testing, one must carefully model econometrically this endogeneity. Anticipating this careful modeling, we propose the following hypotheses that form the basis of a set of empirical tests of the results of our study.

First, in capital intense industries with great economic depreciation, we expect the relation between profitability and investment to be modest and vice versa. Since value maximizing hurdle rates for expansion and maintenance for capital intense industries is relatively low, firms in these industries maintain assets and expand to the extent of their ability to preserve growth option value regardless of whether profitability measured by return on capital is high or low. The relation between profitability and investment is thereby modest.

Second, our study implies a relation between capital intensity, economic depreciation, and industry entry. Figure 1 indicates that the difference between the net-return start threshold for a new business and the net-return maintenance and expansion threshold for an existing business, $\psi^* - \xi^*$, increases with economic depreciation, ρ. Therefore, we expect that in capital intense industries with great economic depreciation, incumbent firms rather than new entrants are responsible for the greatest fraction of incremental industry investment. Maintenance investment creates a barrier to potential industry entrants.

Third, while not directly part of our analysis, our model of maintenance and expansion investment predicts relations between corporate equity returns, profitability, depreciation, and investment. Economic depreciation, profitability, maintenance, and expansion investments all determine each of the so called "earnings response coefficient" from the financial accounting literature[6] which relates stock returns, unexpected corporate earnings changes, and volatility. For example, other things equal, the difference between highest and lowest expected equity returns with respect to corporate profitability – the return on capital – increases with economic depreciation. Maintenance investment creates a "leverage" risk that increases required equity returns. One might use industry depreciation differences to test this hypothesis.

Last, because of lack of guidance from the empirical investment literature, we make the presumption in this paper that maintenance and expansion investments have the same expected net-returns. This could also be seen as a potential limitation of this paper. If this presumption is not confirmed in future empirical testing, then we need to modify our model to accommodate differential net-returns. Of course this possibility is important for guiding managerial investment decisions between maintenance and expansion in economic environments where investment is constrained. This guidance will then generate additional empirical hypotheses on relations between profitability and corporate ancillary investments – maintenance and expansion.

Endnotes

1. Of course, profit is a cash flow based measure.
2. In practice, firms expense those repair and maintenance outlays that do not improve or extend an asset's life. Of course, tax creates an incentive for firms to expense amounts that they might normally capitalize, but we model neither taxes nor tax avoidance.
3. See Kandel and Pearson (2002) for a study of partial reversibility.
4. See Rubinstein (1976) for the economic conditions under which a representative investor can be assumed.
5. At the expense of some additional modeling, one could incorporate multiple stages of growth where, like in the commonly taught Discounted Dividend Model, growth in all but the last stage can exceed both the riskless rate, r, and the risk adjusted rate, r^*.
6. See, for example, Collins and Kothari (1989), Chambers et al. (2005), and Kormendi and Lipe (1987).

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Appendix A

The value of the business, $V(X,B)$, is $V(X,B) = B\pi(Y)$, where $\pi(Y)$ is the value to capital ratio. Let ξ be an arbitrary maintenance and growth gross return hurdle rate. Once we determine the market to capital ratio, $\pi(Y)$, the derivative result $\pi'(Y)_{Y=\xi} \geq 0$ verifies that the manager accepts maintenance and growth investments when gross return exceeds this boundary, $Y \geq \xi$, and otherwise not.

The risk adjusted process for operating profit, X', is:

$$\frac{dX'}{X'} = \begin{cases} -\left(\rho + \theta\sigma_{x,c}\right)dt + \sigma dz, & \text{no maintenance or growth}, Y < \xi \\ \\ (g - \theta\sigma_{x,c})dt + \sigma dz, & \text{maintenance and growth}, Y \geq \xi \end{cases} \tag{A1}$$

Where $\theta \geq 0$ is the coefficient of constant relative risk aversion for a representative investor, σ_{xc} is the covariance of the log of operating profit, *X*, with the log of aggregate consumption in the economy which follows a geometric Brownian motion.

Combining Equation (A1) with Equation (2), and representing the riskless interest rate with the notation, *r*, the market to capital ratio, $\pi(Y)$, satisfies the differential equations,

$$r\pi = \begin{cases} Y - \rho\pi - \theta\sigma_{xc}Y\pi' + \frac{\sigma_x^2}{2}Y^2\pi", & Y < \xi \\ \\ (Y - \rho - g) - \theta\sigma_{xc}Y\pi' + g\pi + \frac{\sigma_x^2}{2}Y^2\pi", & Y \geq \xi \end{cases} \tag{A2}$$

On the upper branch of Equation (A2), there are neither maintenance nor growth expenditures. On the bottom branch, the manager maintains and grows the business, expenditure per dollar of capital is, $\rho + g$. Maintenance offsets economic depreciation, and therefore, profit, *X*, and capital, *B*, grow at a lesser rate, g, than the expenditure rate, $\rho + g$.

The solution to this pair of differential equations, eliminating constants with no economic content, and applying smooth pasting and value matching conditions at $Y = \xi$, is,

$$\pi(Y) = \begin{cases} \frac{Y}{r^* + \rho} + \frac{(\rho+g)\xi}{(r^*+\rho)(r^*-g)}\frac{1-\lambda}{\alpha-\lambda}\left({}^{Y}\!/_{\xi}\right)^{\alpha} - \frac{(\rho+g)}{r-g}\frac{\lambda}{(\lambda-\alpha)}\left({}^{Y}\!/_{\xi}\right)^{\alpha}, & \text{no maintenance, no growth}, Y < \xi \\ \frac{Y}{r^* - g} + \frac{(\rho+g)\xi}{(r^*+\rho)(r^*-g)}\frac{1-\alpha}{\alpha-\lambda}\left({}^{Y}\!/_{\xi}\right)^{\lambda} - \frac{(\rho+g)}{r-g}\left(1 - \frac{\alpha}{(\alpha-\lambda)}\left({}^{Y}\!/_{\xi}\right)^{\lambda}\right), & \text{maintenance, growth}, Y \geq \xi \end{cases} \tag{A3}$$

The parameters α and λ equal,

$$\alpha \equiv \frac{1}{2} + \frac{\theta\sigma_{xc}}{\sigma^2} + \sqrt{\left(\frac{1}{2} + \frac{\theta\sigma_{xc}}{\sigma^2}\right)^2 + \frac{2(r+\rho)}{\sigma^2}} \geq 1$$

$$\lambda \equiv \frac{1}{2} + \frac{\theta\sigma_{xc}}{\sigma^2} - \sqrt{\left(\frac{1}{2} + \frac{\theta\sigma_{xc}}{\sigma^2}\right)^2 + \frac{2(r-\rho)}{\sigma^2}} \leq 0 \qquad \text{(A4)}$$

The first term on the right hand side of the upper branch of Equation (A3) is the value of a permanently non-growing business with unmaintained assets. The second term is the value of the option to start maintenance and growth after periods of non-growth and maintenance neglect, indefinitely and at any future time. The third term is the cost of maintenance and growth investments during period of maintenance and growth when they arise in the future.

The first term on the right hand side of the lower branch of Equation (A3) is the value of a permanently maintained and growing business. The second term is value loss during future period of maintenance-neglect and non-growth. The third term is the cost of maintenance and growth recognizing that there will be future periods of maintenance-neglect and non-growth.

Set the derivative of (A3) with respect to ξ, on either branch, to zero, and evaluate at $\Upsilon = \xi$, to find the value maximizing maintenance and growth hurdle rate that we report in Equation (4).

Appendix B

In this appendix, we proof that the value maximizing net-return boundary, $\xi^* - \rho$, in Equation (4) decreases with economic depreciation, ρ. First, we establish that the value to capital ratio in Equation (A3) is one at the value maximizing gross return boundary, ξ^*, in Equation (4), $\pi(\xi^*) = 1$. Substitute the expression for ξ^* on the right hand side of Equation (4) into either branch of Equation (A3):

$$\pi(\xi^*) = \frac{\lambda(1-\lambda)(\rho+g) + (r^* - g)\alpha\lambda(\alpha-\lambda)}{(r-g)(\alpha-1)(\lambda-1)(\alpha-\lambda)}$$

This expression equals one if:

$$r(\lambda-\alpha) + r(\alpha^2-\lambda^2) + \alpha g(1-\alpha) + \alpha^2\lambda\theta\sigma_{xc} - \alpha\lambda^2\theta\sigma_{xc} + \rho\lambda(1-\lambda) = 0$$

Substitute the definitions for α and λ to verify this equation.

Take the derivative of Equation (4),

$$\frac{d\ \xi^* - \rho}{d\rho} = \frac{(r^*-g)\alpha\lambda}{(r-g)(1-\alpha)(1-\lambda)} + \frac{(r^*+\rho)(r^*-g)\lambda}{\sigma^2(r-g)(1-\alpha)^2(1-\lambda)\sqrt{\frac{2(r+\rho)}{\sigma^2} + \left(\frac{1}{2} + \frac{\theta\sigma_{xc}}{\sigma^2}\right)^2}} - 1 \quad \text{(A5)}$$

Because the value to capital ratio equals one at the value maximizing maintenance and growth hurdle rate, $Y = \xi^*$,

$$\frac{(r^*-g)}{(r-g)}\frac{\alpha}{(1-\alpha)}\frac{\lambda}{(1-\lambda)} + \frac{(\rho+g)\lambda}{(r-g)(\alpha-\lambda)(1-\alpha)} = 1 \qquad \text{(A6)}$$

Substitute (A6) into (A5). Then, $\frac{d\ \xi^* - \rho}{d\rho} \leq 0$ if,

$$\frac{(\rho+g)}{(\alpha-\lambda)} \geq \frac{(r^*+\rho)(r^*-g)}{\sigma^2(1-\alpha)(1-\lambda)\sqrt{\frac{2(r+\rho)}{\sigma^2} + \left(\frac{1}{2} + \frac{\theta\sigma_{xc}}{\sigma^2}\right)^2}}$$

This inequality always holds because the left hand side is positive, whereas, the right hand side is negative.

Appendix C

The value to book ratio prior to new business start, say, $\eta(Y)$, satisfies,

$$r\eta = -\theta\sigma_{x,c}Y\eta' + Y^2\eta"\sigma^2/2$$

The solution to this differential equation, eliminating inappropriate constants, is,

$$\eta(Y) = cY^{\varpi} \qquad \text{(A7)}$$

Where c is an arbitrary constant and,

$$\omega \equiv \frac{1}{2} + \frac{\theta\sigma_{xc}}{\sigma^2} + \sqrt{\left(\frac{1}{2} + \frac{\theta\sigma_{xc}}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} \geq 1 \qquad \text{(A8)}$$

Determine ψ^* and the arbitrary constant, c, with the following two (value matching and smooth pasting) conditions,

$$\eta(\psi^*) = \pi(\psi^*) - 1$$
$$\eta'(\psi^*) = \pi'(\psi^*)$$

Determine the result in Equation (6) with these two equations, Equation (A7), and the lower branch of Equation (A3).