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Multifractal Analysis of the Algerian Dinar - US Dollar Exchange Rate

Sami Diaf* and Rachid Toumache**

This article aims to study the scaling behavior of the Algerian Dinar-US Dollar exchange rate using multifractal time series analysis which stems from the fractal theory first implemented by Benoît Mandelbrot in early 1960. Investigating time series properties using this technique permits to shed light on important characteristics omitted by traditional time series analyses and highlight the usefulness of local Hölder exponents in predicting crash patterns, as well as identifying the nature of the evolving shocks affecting the series.

JEL Codes: C5, C22 and F31

1. Introduction

The behavior of the exchange rate has been of a crucial concern for both economists and decision makers which rely on this macroeconomic variable when building up their economic policies and the associated forecasts. In this time of economic and financial turmoil, the exchange rate is of a great importance in the worldwide economic debate, where many countries try to operate depreciative adjustments to have a favorable international competitiveness. Many studies dealt with that topic to better assess this behavior and its impact on an eventual internal/external shock.

Most of the studies related to exchange rate assessments deal with the class of real exchange rate, to test the hypothesis of the purchasing power parity (PPP) and to conclude whether a series of real exchange rate follows a random walk or, instead, the PPP hypothesis holds in the long run. Testing the PPP hypothesis using unit roots has been widely criticized as researchers could not distinguish between a random walk behavior and a slow mean reversion process (Caporale and Gil-Alana 2010). To address such imperfections, analysts used long-memory methods and fractional processes and found, for some series, positive memory during a period of flexible exchange rate.

The second class of exchange rate models focus on the nominal exchange rate and aims at determining models providing the best predictions of financial volatility, rather than its determination.

Hence, most academic papers were based on the Efficient Market Hypothesis (EMH) which grew in popularity since early 1970 (Fama 1970). It supposes the price of an asset at the instant t contains all past information until $t-1$.

* Sami Diaf, PhD candidate in Economics and Applied Statistics, High National School of Statistics and Applied Economics (ENSSEA, Algiers). Email: sami.diaf@yahoo.com
** Dr. Rachid Toumache, Assistant Professor of Economics and Senior Researcher, High National School of Statistics and Applied Economics (ENSSEA, Algiers). Email: rtoumache@gmail.com

In its weak form, this hypothesis implies that the price of an asset, which follows a log-normal distribution, obeys to a martingale process (random walk), whereas the returns in logarithms are independent and identically distributed.

Peters (Peters 1994) proposed the Fractal Market Hypothesis (FMH) approach based on market stability rather than its efficiency, given the EMH could not predict different economic crises and stock market crashes occurring since that time (Onali and Goddard 2009). This approach led to a new fractal dimension-based models which performed better than traditional linear models because of their ability to deal with the non-linearity characteristics of financial series (Mandelbrot and Hudson 2004).

For the case of Algeria, as for many developing countries, the exchange rate serves as the basis of the economic policy and the budget management, bearing in mind its status of single-commodity exporter (hydrocarbons), getting 98% of its revenues in US dollar and trading up to 60% with its partners using the single currency (Euro). Hence, the Euro-US Dollar parity has a significant impact on the country's macroeconomic equilibrium.

Works on the nominal exchange rate in Algeria targeted the behavioral side, stressing the use of classical linear time series analysis for that purpose because of the lack of low-frequency macroeconomic data. These traditional techniques are limited in their application and cannot address the nature of shocks affecting the series.

We propose in this paper to use the Fractal Market Hypothesis (FMH) and the underlying fractal approach to examine the scaling behavior of the nominal exchange rate series of the Algerian Dinar - US Dollar. This approach stems from the fractal theory first implemented by Benoît Mandelbrot, being widely used nowadays in different scientific disciplines. Focus will be on studying the nature of the Hurst exponent, which is derived from the fractal dimension, and determining its local properties.

This aim of this paper is to study, for the first time, the scaling properties of the Algerian Dinar–US Dollar exchange rate using daily data over 16 years. It is organized as follows. Section 2 gives a theoretical foundation of the Hurst exponents and the methods, basic and general, to estimate them. Section 3 presents the empirical application and the scaling properties of the series, with a stress to their local variations using Hölder exponents.

2. Theoretical Foundations

The fractal theory in time series analysis was linked first with the *Rescaled Range Analysis*, a method implemented by British hydrologist Harold Edwin Hurst when studying the Nile river's flood. It results the Hurst statistics (Hurst exponent) which determines how the memory process behaves. For $0<H\leq 0.5$: the series is said to have a short-memory with anti-persistent shocks, while for $0.5<H<1$ we observe a long-memory phenomenon. Hence, the fractal dimension D, a statistic measuring the degree of roughness of a given series, is obtained by the relationship $D = 2 - H$.

In general, the FMH is based on market structure. Applying the self-similarity concept on a series of returns $X(t)$, we define Hurst exponent (H) by $X(ct) = c^H X(t)$. This relationship defines a unique H, unchanged over time known as the unifractal case, while a more general relationship (Mandelbrot and Fisher and Calvet 1997) defines the multifractal case $X(ct) = M(c)X(t)$ where $M(c)$ is a function of H.

Traditional methods estimating H consist of estimating, for a given interval, the ratio between the range R of the centered integrated series, and the standard error S of the original series. A particular interest is given to the difference between the minimum and the maximum of the deviations, in a certain interval, normalized by the standard error.

The Hurst exponent is reckoned as follows:

1. Calculate the mean (m) of the series X_i i=1…n
2. Generate the centered series $Y_i = X_i - m$ for i=1…n
3. Generate the series of cumulative deviations $Z_i = \sum Y_i$
4. Calculate the range $R(n) = \max(Z_1, Z_2, \dots, Z_n) - \min(Z_1, Z_2, \dots, Z_n)$
5. Calculate the standard deviation : $S(n) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(X_i - m)^2}$
6. Calculate the rescaled range series: $\frac{R(n)}{S(n)}$ and the means on each n.
7. The Hurst exponent is obtained via the power law: $E\left(\frac{R(n)}{S(n)}\right) = C.n^h$, so we regress the logarithm of $E\left(\frac{R(n)}{S(n)}\right)$ over log(n), and we obtain the slope which serves as an estimator of H.

This classical method is widely used, especially in hydrology, for series supposed to have a unifractal (monofractal) characteristic, say a unique constant exponent H over time. Its principal inconvenient lies on its sensitiveness for short time series having less than 500 observations.

Many time series, especially financial ones, are non-stationary and have a volatile behavior leading to a variable H, hence a multitude of Hurst exponents for different time scales (multifractal case). This approach is more indicated for series presenting high volatility, switching regimes and sudden shocks.

This idea recalls another classic alternative to the rescaled range analysis which is the multifractal detrended fluctuation analysis (MF-DFA) that aims to detect self-similarity processes in a time series. This is done via the following steps (Kantelhardt 2008):

1. Given the initial series x_i, calculate the cumulative series $Y(j) = \sum_{i=1}^{j} x_i$
2. Divide the series $Y(j)$ into $N_s = int(\frac{N}{s})$ independent segments of the same length. Because the length of the series N is usually not a multiple of the time scale s, we can repeat the procedure by starting from the opposite side (end of the series).We will have hence, $2N_s$ segments.
3. Calculate the local polynomial of each of the $2N_s$ segments via an ordinary least square regression of the profile $Y(j)$ on each segment $\upsilon = 0, \dots, 2N_s - 1$. We can also use high order polynomial and the method is then called *MF-DFAm* where m is the adjustment order. In our case, we set m=2 and $F^2_{DFAm}(\upsilon, s) = [Y(\upsilon s) - Y((\upsilon + 1)s)]^2$ is the square root of the mean deviation of a random walk after s steps (segments).
4. We calculate the q-th order fluctuation function$F_q(s) = \left\{\frac{1}{2N_s}\sum_{\upsilon=1}^{2N_s}[F^2_{DFAm}(\upsilon, s)]^{\frac{q}{2}}\right\}^{\frac{1}{q}}$ and we focus on the relationship between the variations of q and $F_q(s)$. The steps 2, 3 and 4 must be repeated over several steps s and $F_q(s)$ is only defined for $s \geq m + 2$.

5. We determine the scaling behavior of the fluctuations functions by log-regressing $F_q(s)$ over s for several values of q. $F_q(s) \sim s^{h(q)}$ where $h(q) = \frac{1+\tau(q)}{q}$ and $\tau(q)$ is called the Renyi exponent. By using a Legendre transform, we determine the degree of singularity or the Hölder exponent $\alpha = h(q) + q * h'(q)$ and deduce the singularity spectrum $f(\alpha) = q[\alpha - h(q)] + 1$.

The statistic $F_q(s)$ becomes unreliable for $s \geq N/4$ and the value of $h(0)$ corresponding to the limit of $h(q)$ for q approaching 0 cannot be obtained directly. Will use a logarithmic procedure: $F_0(s) = exp\left\{\frac{1}{4N_s}\sum_{\upsilon=1}^{2N_s} ln[F^2(\upsilon, s)]\right\} \sim s^{h(0)}$.

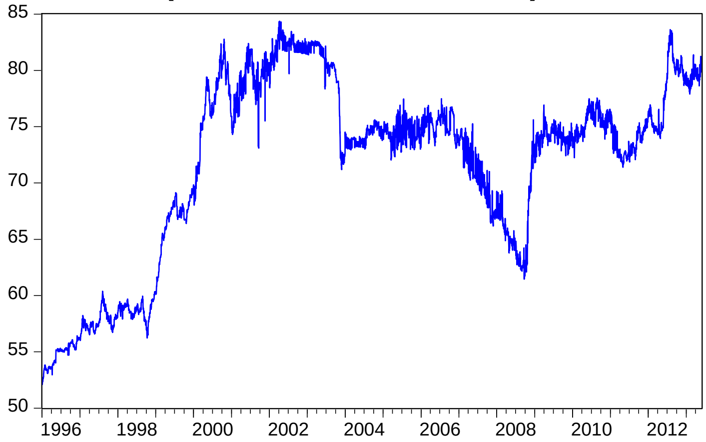
In the monofractal case, $h(q)$ is independent of q because the function $F^2_{DFAm}(\upsilon, s)$ is identical for each segment υ. Hence, for positive values of q, the segments υhaving a large variance $F^2(\upsilon, s)$have a tendency to dominate the mean $F_q(s)$ and $h(q)$will describe the scaling behavior of segments having large fluctuations (the opposite remains true).

The Hölder exponent (1) has a particular importance in the multifractal analysis (Cheng and Huang 2011) as it brings into evidence the infinitesimal variation of a given exponent around an instant t. Hence, it will be used to study local variations of Hurst exponents.

3. Application

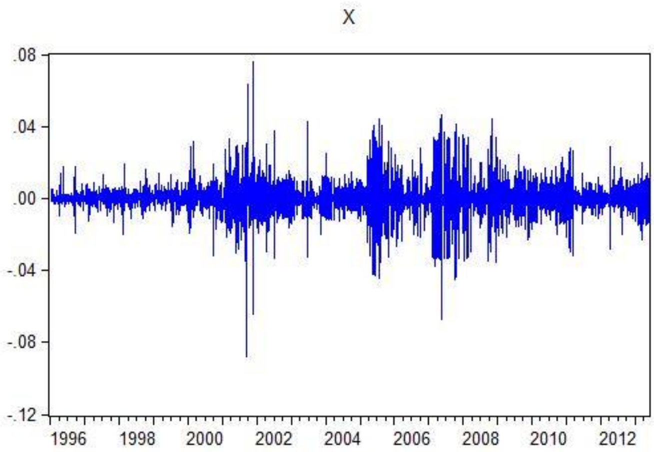
We consider the daily series of Algerian Dinar - US Dollar exchange rate during the period January 1996 to end-May 2013 (6,362 observations) The start of the series corresponds to the new quotation system for the Dinar (base 1995) based on a basket of 15 foreign currencies representing the then-trading partners, as a result of applying the International Monetary Fund's structural adjustment program (1994-1998). Figure 1 shows different periods of appreciation/depreciation of the Dinar according to the prevailing economic situation. These movements were linked to the good performance of international oil prices, given the position of Algeria, a single-commodity exporter (hydrocarbon), and the impact of the Euro/US Dollar parity.

Figure1: Evolution of the Algerian Dinar - US Dollar exchange rate, raw data (01/01/1996 – 31/05/2013).



We will analyze the scaling behavior by using the logarithm difference of the exchange rate $r_t = \log(X_t) - \log(X_{t-1})$ where X_t is the series of the daily exchange rate.

Figure 2: Evolution of the Algerian Dinar - US Dollar exchange rate, in logarithm difference (01/01/1996 – 31/05/2013).



The raw series is highly volatile and non-stationary, but the logarithm difference returns a stationary series with many clusters of volatility.

Table 1: Descriptive statistics of the series in logarithm difference

Mean	Median	Std. Dev.	Range	Skewness	Kurtosis
$6.68 * 10^{-5}$	0.000	0.0073	0.1641	-0.1587	18.9706

Figure 3 displays of the fluctuation functions $F_q(n)$ for a scale $16 < n < 1024$, given different values of q={-5,0,+5}, show that adjustment lines are neither superposed nor parallel, meaning many Hurst exponents could be present in the series due to possible switching regimes. By retrieving the values of H for different values of q (Figure 4), it appears clearly the function $H(q)$ is linear, decreasing but unstable, which is an evidence of the multifractal character of the series (2). Moreover, the singularity spectrum shows a multifractal spectrum with a range of 0.4596 which corresponds to the difference of maximums of $H(q)$ (Figure 5 and 6). This spectrum should be null in the case of monofractality (Ihlen 2012).

Figure 3: Evolution of F_q in function of segments s.

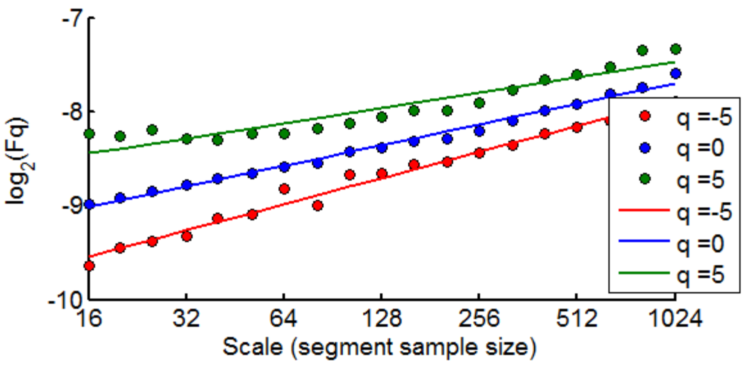


Figure 4: Evolution of H_q in function of moments q.

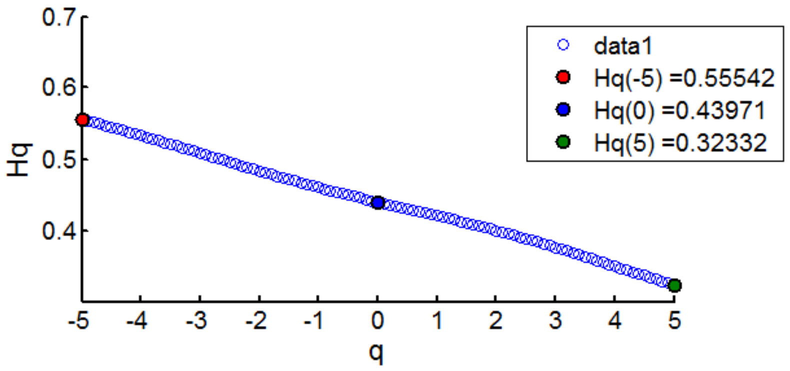


Figure 5: Evolution of $\tau(q)$ in function of moments q.

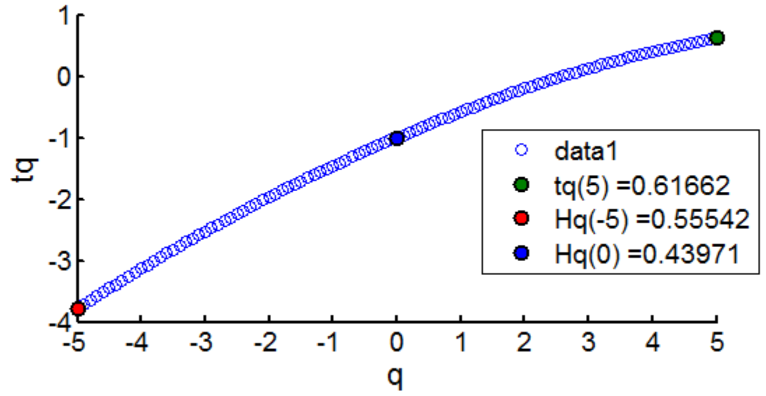
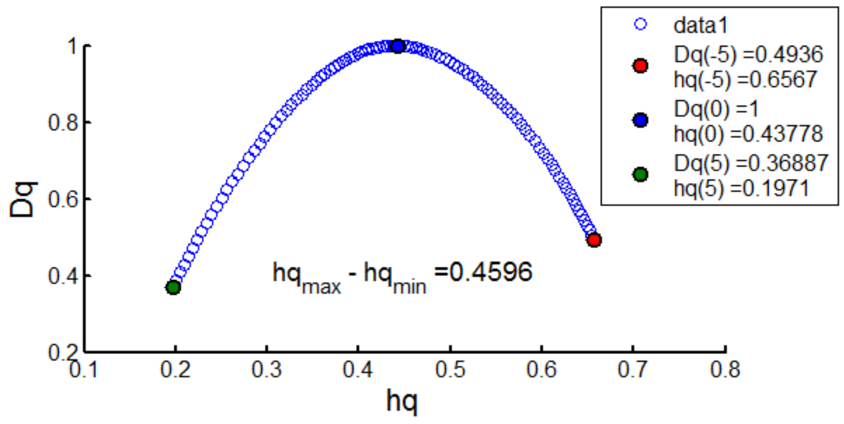


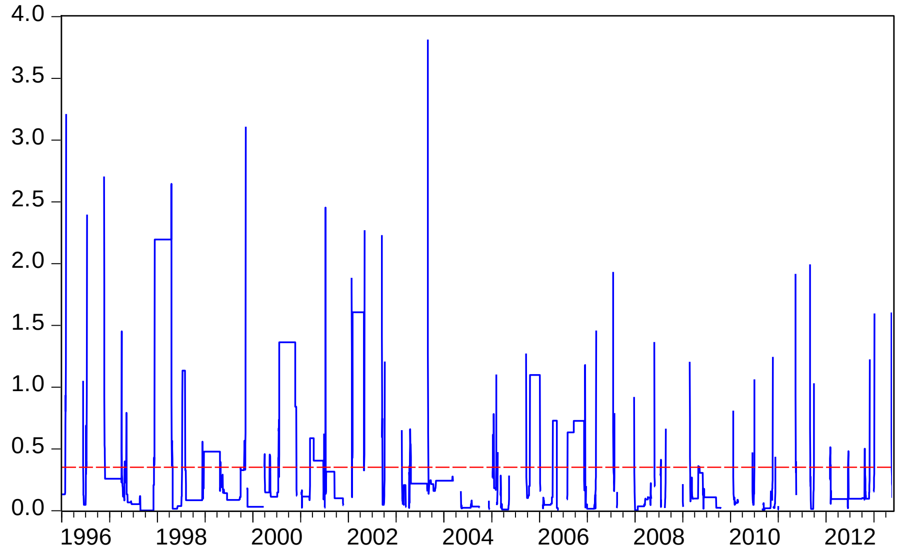
Figure 6: Evolution of the multifractal spectrum and its range.



Recent developments of the multifractal analysis (3) permit to determine critical moments, or crash patterns, of a given series by studying its local regularity via local Hölder exponents. A Hölder exponent (in the pointwise estimation) comprised between 0 and 1 means the signal is continued but non-differentiable at a given point and a low value of that exponent indicates an irregularity in the signal. A critical event is preceded by a sudden increase of the regularity exceeding the unity, followed by small values of regularity for a long period (Agaev and Kuperin 2004).

Figure 7 indicates clearly the series present crash patterns in different intervals (Véhél and Legrand 2004), linked to sharp appreciations/depreciations of the Dinar, given the prevailing economic context. Thus, Figure 8 gives us a clearer view by decomposing the series in distinct intervals for the sake of clarity.

Figure 7: Evolution of local Hölder exponents (pointwise estimation) of the series in logarithmic difference (the dashed red line represents the mean of Hölder exponents).

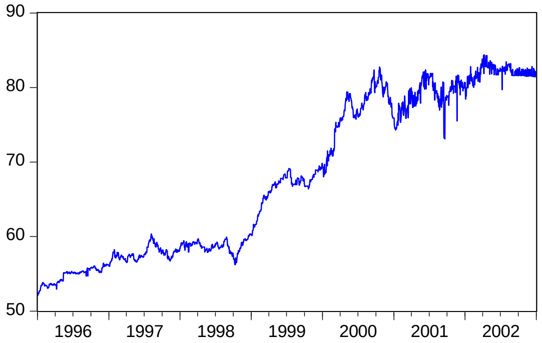


We notice local Hölder exponents' peaks preceding prolonged depreciations/appreciations of the Dinar, in other terms; they are harbingers of the series' trend breaks, even if there are clusters volatility. This is the case of the continued depreciation (21.26%) during the period mid-October 1998 to mid-July 1999, followed by another depreciation of 19.21% (mid-October 1999 to earl-May 2000). The crash pattern determines also the sharp volatility of the exchange rate occurred during the second semester of 2001.

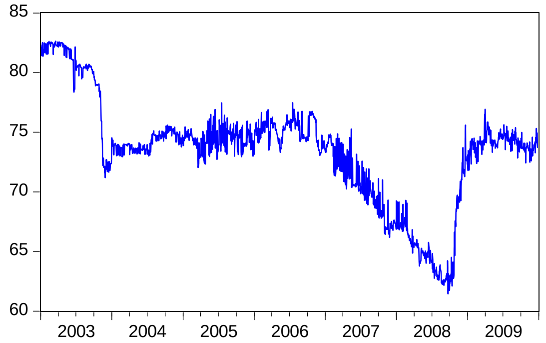
Another event detected by local Hölder exponents is the brutal appreciation of the Dinar (11%) observed from September 2003 to November 2003, the volatile but sustained appreciation from February 2007 to September 2008 (16%) and the following depreciation of the similar amplitude (18%), which brought the exchange rate to its level reached 2 years ago. Finally, we find less pronounced variations in amplitude during the period early 2010-May 2013, but conserving a general depreciative trend.

These evolutions are correlative from one side to the good tenure of international oil prices and to the external financial position of Algeria, from another side. This translates the authorities' commitment to keep a favorable external position to face potential external shocks and international crises.

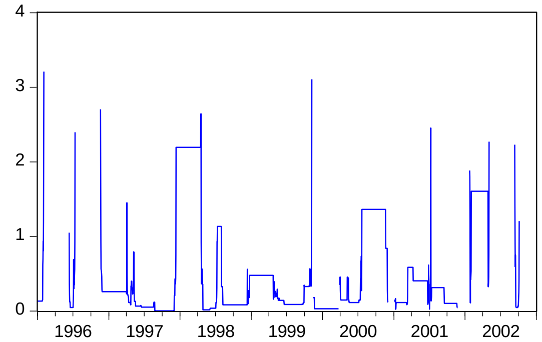
Figure 8: Evolution of the Algerian Dinar – US Dollar exchange rate and the local Hölder exponents (pointwise estimation) for the periods 1996-2002, 2003-2009 and 2010- May 2013.



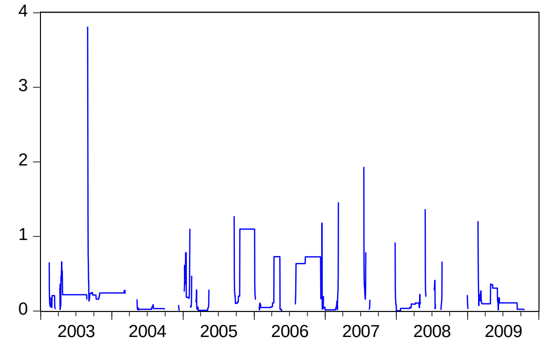
The exchange rate series Algerian Dinar - US Dollar (1996-2002)



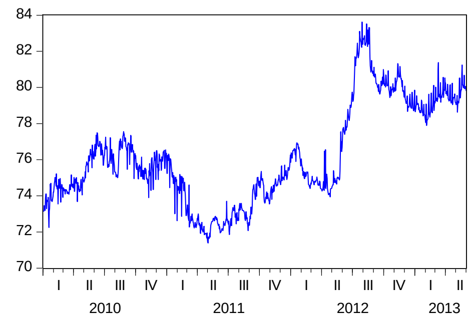
The exchange rate series Algerian Dinar - US Dollar (2003-2009)



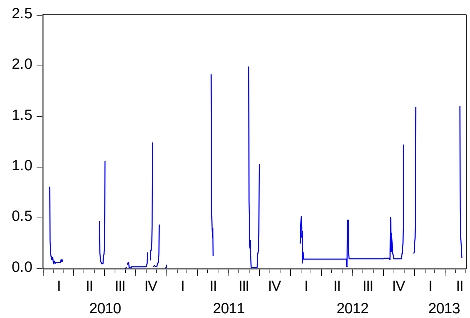
Series of local Hölder exponents (1996-2002)



Series of local Hölder exponents (2003-2009)



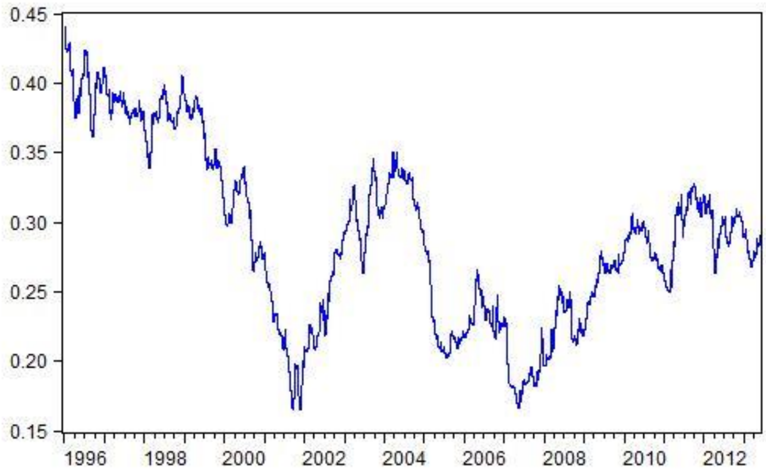
The exchange rate series Algeria Dinar - US Dollar (2010-2013)



Series of local Hölder exponents (2010-2013)

Another interesting evidence is given by Local Hölder exponents obtained via oscillation-based method (Figure 10) showing the local Hurst exponents have their values ranging between 0.16 and 0.44. Thus, the spectrum exhibits a variable anti-persistence pattern, confirming the process is mean-reverting.

Figure 9: Evolution of local Hölder exponents(oscillation-based method) of the series in logarithmic difference.



4. Conclusion

This paper analyzed the daily series of the Algerian Dinar-US Dollar exchange rate during the period January 1996 to May 2013 and proved, using the Multifractal Detrended Fluctuation Analysis, the multifractality of the series and confirmed the presence of a spectrum of Hurst exponents with local values ranging from 0.16 to 0.44. This analysis departs from the standard rescaled range analysis, which computes a single Hurst exponent for the whole series, and emphasizes the power law relationship between the absolute moments of returns and the sampling intervals.

This empirical investigation demonstrates that the series is subjected to a variable anti-persistent regime, displayed by different episodes of currency appreciation/depreciation due to oil prices' volatility and efforts to guarantee a stabilization of the real effective exchange rate via the adoption of a reactive monetary policy. These findings serve to calibrate fractional processes in a way they take into account the mean-reverting characteristic exhibited by the series.

Multifractal properties were reinforced by the analysis of local Hölder exponents which displayed several critical periods that have great macroprudential significance for policy makers since they serve as harbingers of potential critical events or turbulence periods that could influence the behavior of the Algerian Dinar.

Endnotes

(1) A function $f(x)$ belongs to the α-order class of Hölderian functions if $|f(t+h) - f(t)| < const.h^{\alpha}$ where $t, h \in \mathbb{R}$ and $0 \leq \alpha \leq 1$. In the case where α depends on t $(\alpha \rightarrow \alpha(t))$ then $\alpha(t)$ is said to be local Hölder exponent.
Hence, the local Hölder exponent is deduced via the relation: $|LnX(t+\Delta t) - LnX(t)| \sim C_t(\Delta t)^{\alpha(t)}$ where C_t is a time prefactor. The unifractal case occurs when $\alpha(t) = H$ is constant over time.

(2) *Matlab* codes used for this study were provided by E.A.F Ihlen (Ihlen, 2012).
(3) Pointwise (local quadratic variations) and oscillation estimation of local Hölder exponents were executed using Fraclab, a Matlab toolbox developed by the research team of Professor Jacques-Lévy Véhel at the French Institute for Research in Computer Science and Automation (INRIA).

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