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Fertility Choice by the Couples: A Multinomial Model for Household Decision Making

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This paper formulates a micro-founded and optimization based choice model where husband and wife make inter-temporal fertility decisions in a life cycle to optimize utilities and household's wealth subjecting to financial and health constraints, specifically, husband and wife make rational and dynamic decisions about the number of children to produce and the expenditure on children to maximize household's wealth over life span. Gender differences in decision making about fertility choice and wealth accumulation are captured by specifications of utility functions, preference parameters and income constraints. Hypothesis concerning whether husband and wife's optimal fertility decisions differ in direction and magnitude due to heterogeneous utility functions, household income, inflation and child welfare policies is tested. The methodologies involve random utility discrete choice models governing a couple's behavior from the theoretic perspective and linear regression estimation with parametric distributions of random coefficients from the empirical perspective. Panel data including Australian birth rates, household income, female/male earnings, maternity leave payment, mortality rate, child-care subsidies, education expenses, living costs indices and household expenditure on health are collected to test the theory empirically. The conclusions are that the optimal number of children and expenditure on children are jointly determined by risk aversion coefficients of utility functions, household income, child care expenses and subsidies. The choice to produce the first baby is a balanced outcome from the interaction among the dynamics of household income, number of marriages, maternity payment, difference in men and women's earnings, mortality rate and the number of women who worked after child birth. The choice to produce multiple babies is influenced additionally by decreasing marginal child care cost and congestion cost in raising multiple children. The choice of expenditure on raising children is more flexible and dynamic, couples make a series of optimal choices in accordance with each period's child-care subsidies, education expenses and living costs indices.

1. Introduction

Whether husbands and wives systematically differ in their attitudes and responses towards risk is an important economic and psychological question. Husbands and wives, who are subject to heterogeneous biological constraints and distinct social expectations, usually behave differently in making career decisions, financial decisions, household decisions and fertility decisions. These psychological and behavioural heterogeneities, which are mainly caused by gender difference, not only influence individual households' fertility decision, labour supply decision and wealth accumulation directly, but also impact upon aggregate economy's population growth, labour force participation and productivity growth indirectly.

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These motivate scholars to investigate the distinct preferences between males and females, the interaction mechanisms between husbands and wives, and the fundamental connections between financial decisions and fertility decisions. Previous researchers have formulated a variety of household decision making models; these models not only distinguish gender difference in preference and risk aversion, but also characterize the linkage between financial decision making process and fertility decision making process. However, the majority of previous researchers do not model the household couples' decision making process interactively by accounting for husbands' expectations about wives' behaviour and wives' expectations about husbands' behaviour. Besides, the majority of household models do not characterize the couples' fertility decision variables and infertility risk dynamically in a probability setting. This paper intends to fill in these research gaps by formulating a dynamic, stochastic and expectational household decision making model subject to a series of financial constraints and biological constraints.

The paper is organized as the following. The first part reviews the literature in modelling gender differences and households' fertility decision making. The second and the third parts involve theoretical modelling and empirical analysis. The final part concludes the main findings.

2. Literature Review

Males and females have different motivations and perceptions in household decision making. Gender differences lie in the perception of dividing low-control and high-control household tasks between a couple, the power and the speed of making household decisions. Bartley, Blanton and Gilliard (2005) examine differences in perceived decision-making, gender-role attitudes, division of household labour, perceived marital equity in dual-earner husbands and wives, the impact of perceived decision making and gender-role attitudes, and they find that decision-making, low-control household labour and high-control household labour differ significantly between husbands and wives, wives spend more time in household labour and are much more likely to be involved in low-control household tasks. Becker, Fonseca-Becker and Schenck-Yglesias (2006) assess husband's and wife's reports of decision-making on items purchase, child rearing and medical care in a household, they find that wives tend to under-report their household decision-making power, women's reports of their decision-making power are significantly related to the household having a plan for what to do in case of a maternal emergency, but are not associated with place of childbirth or with having a postpartum check-up, while husband's reports of the wife's decision-making power are negatively associated with the likelihood of having the last birth in a health facility. Reiter (2013) tests the gender differences in decision-making patterns when multiple options were available, and he finds that when males have formed a routine and are in their more natural environment, they are quicker to make a decision than females, but when in a less familiar situation, males and females do not significantly differ in the amount of time it take to make decisions.

Males and females have different degrees of risk aversion. Women are stereotyped as more risk averse than men, researchers have integrated gender difference in the specification of utility function and decision making process in the literature. Powell and Ansic (1999) examine gender differences in risk propensity and strategy in financial decision making, and they find that females are less risk seeking than males irrespective of familiarity and framing, costs or ambiguity. Powell, Schubert and Gysler (2001) integrate gender aspects into expected utility model, rank dependent expected utility model, prospect theory, security potential model and risk/return model for decision theory, they summarize that women's utility functions are more concave than men's utility functions since risk aversion is measured by the curvature of the utility function, and that women are more likely to devalue the probabilities of positive outcomes since risk aversion is also measured by the elevation of the probability weighting function. Eckel and Grossman (2008) review the results from experimental measures of risk aversion for evidence of systematic differences in the behaviour of men and women, they find that in most studies, women are found to be more averse to risk than men and studies with contextual frames show less consistent results.

Males and females' household decision making behaviours are influenced by a number of factors. Iyigun and Walsh (2002) present a microeconomic model of the household under which there exists no difference in spousal preferences but where childrearing is more time costly for women and indicated that bargaining between wife and husband forms the basis of household decisions, they find that marital bargaining power is determined endogenously according to the relative labour income of the spouses, they reveal that empowering women through institutional reforms lends to lower fertility and higher educational attainment, and they show that improvements in life expectancy and a lower gender wage gap empower women and divert household resources to education. Lawrence (2003) examines gender inequality in household decision-making and how household decision-making evolves with time, she finds that household behaviour and preferences evolve over time with increasing opportunity cost of women that accompanies technological change, better educational opportunities and social norms, that household behaviour progresses from one where husband dominates decision-making to one where decision-making is more egalitarian, and that increased job opportunities, higher income or increased education of women would be expected to result in better family health, nutrition and lower fertility rates. Neyer, Vignoli and Lappegård (2012) examine to what extent a couple's employment opportunities, financial situation and care options matter in the decision making process and whether the anticipated consequences on these aspects affect the intention to have a child in the next three years, they find that women and men who expect that having a child will have negative consequences on their employment or the financial situation of their family are less inclined to have a child than those who do not expect negative consequences on their employment or financial situation, and they also find that those whose decision to want a child in the next three years depends heavily on the availability of childcare are less inclined to intend to have a child than those who put less weight on the availability of childcare.

However, previous researchers focus on examining gender difference in the power and speed of decision making process, detecting husbands and wives' difference in risk aversion, and explaining the biological, financial and education factors' influence mechanism upon households' decision making. These findings, however, do not elucidate explicitly how couples' biological, psychological, educational and financial attributes impact upon their joint fertility choices, how couples' fertility decision making process are influenced by their biological constraints and financial constraints over time. This paper will overcome these limitations by establishing new types of household fertility decision making models theoretically and empirically.

3. Theoretical Model

The theoretical model inherits and improves previous models in the literature by formulating a dynamic, probablistic and expectational household decision making model subject to a series of financial constraints and biological constraints. Assume husband and wife are of the same age, form up a family and start to work at the age of 18, retire at the age of 66 and die at the age of 100, both begin to make fertility decisions each year from the age of 18 to the age of 50, raise their children together by contributing to children's consumption and education expenditure in terms of both money and labour, as long as the children survive and wish to stay in the household, the parents will keep them until the parents' or children's death.

3.1 Husband's and Wife's Utility Functions and Risk Aversion

Husband's life time utility function is specified as the summation of discounted utility of consumption $C_{Husband,t}$, the average quality of each survived child Q_t and the total number of survived children N_t at each period t, while wife's life time utility function is specified as the summation of discounted utility of consumption $C_{Wife,t}$, the average quality of each survived child Q_t and the total number of survived children N_t at each period t.

$$U_{Husband} = \sum_{t=18}^{100} \beta_{Husband}^{t-18} U_{Husband}(C_{Husband,t}, Q_t, N_t) = \sum_{t=18}^{100} \beta_{Husband}^{t-18} \left(\frac{{C_{Husband,t}}^{1-\alpha_{Husband}}}{1-\alpha_{Husband}} + \frac{(Q_t \cdot N_t)^{1-\alpha_{Husband}}}{1-\alpha_{Husband}} \right)$$

$$U_{Wife} = \sum_{t=18}^{100} \beta_{Wife}^{t-18} U_{Wife}(C_{Wife,t}, Q_t, N_t) = \sum_{t=18}^{100} \beta_{Wife}^{t-18} \left(\frac{{C_{Wife,t}}^{1-\alpha_{Wife}}}{1-\alpha_{Wife}} + \frac{(Q_t \cdot N_t)^{1-\alpha_{Wife}}}{1-\alpha_{Wife}} \right)$$

Where $\beta_{Husband}$ and β_{Wife} are the discount factors of husband and wife respectively, Borghans, Golsteyn, Heckmand and Meijers's (2009) investigate gender difference in decision making and find that women are more risk averse than men, since wife as a female is assumed to be more risk averse than husband as a male, wife's discount factor β_{Wife} is smaller than husband's discount factor $\beta_{Husband}$ in terms of $\beta_{Wife} < \beta_{Husband}$; both husband's and wife's utility functions take the form of constant relative risk aversion utility functions, $\alpha_{Husband}$ and α_{Wife} are the relative risk aversions for husband and wife respectively, since relative risk aversion of husband equals the constant $\alpha_{Husband}$i while relative risk aversion of wife equals α_{Wife}ii; $\frac{1}{\alpha_{Husband}}$ and $\frac{1}{\alpha_{Wife}}$ are the elasticity of intertemporal substitutions for husband and wife respectively, since wife is assumed to be more risk averse than husband, $\alpha_{Wife} > \alpha_{Husband}$, wife is less elastic of intertemporal substitutions in terms of $\frac{1}{\alpha_{Wife}} < \frac{1}{\alpha_{Husband}}$.

3.2 Number Survived and Dependent Children

Inspired by Sah's (1991) specification of the binomial density of survived children in a single-stage model of fertility choice and Sommer's (2014) specification in the law of motion of dependent children in a household, assume the number of survived children in a household follows the following process: $N_t^B = N_{t-1}^B + K_t$ where N_t^B is the number of children produced by husband and wife by the time t; K_t is the fertility decision model, K_t only has two values 1 and 0, K_t equals 1 with probability g when husband and wife produces a child at period t, K_t equals 0 with probability 1-g when husband and wife produces a child at period t, 0<g<1, the behaviour of K_t is modelled by the following binary logistic regression model.

Table 1: Probability Density of Fertility Decision Variable K_t

Fertility Decision Variable K_t	1 (produce 1 child)	0 (do not produce a child)
Probability	g	1-g

$$K_t = \begin{cases} 0 & 1 \le t < 18 \\ \mathrm{Ln}\left(\frac{g_t}{1-g_t}\right) & 18 \le t \le 50 \\ 0 & 50 < t \le 100 \end{cases}$$
$$= \begin{cases} 0 & 1 \le t < 18 \\ \beta_0 + \beta_1 \cdot W_{Husband,t} + \beta_2 \cdot W_{Wife,t} + \beta_3 \cdot S_t + \beta_4 \cdot H_{Husband,t} + \beta_5 \cdot H_{Wife,t} + u_t & 18 \le t \le 50 \\ 0 & 50 < t \le 100 \end{cases}$$

Where $W_{Husband,t}$ and $W_{Wife,t}$ are the wages of husband and wife respectively at time t, S_t is government subsidy of one single child care at time t, $H_{Husband,t}$ and $H_{Wife,t}$ are the general health conditions at time t, u_t is the disturbance term at time t.

$$N_t = \sum_{j=18}^{t} K_j \cdot I_{t-j<18} \sim B(N_{t-1}^B + K_t, S)$$

Where the indicator variable $I_{t-j<18} = \begin{cases} 1 & t-j < 18 \\ 0 & t-j \ge 18 \end{cases}$, B represents a binomial distribution with the two parameters in terms of total number of children $N_t^B = N_{t-1}^B + K_t$ produced by husband and wife and the survival rate S of each child at period t, 1-S is the mortality rate of children at period t, 0<S<1, assume both S and 1-S are exogenous.

3.3 Quality of Survived and Dependent Children

Following Sommer's (2012) specification, assume each child is of the same quality and each child's quality function takes on the following constant elasticity of substitution form.

$$Q_t = Q\left(C_{Child,t}, E_t, L_{Husband,t}, L_{Wife,t}, N_t\right) = \left[\gamma_1 \cdot \left(\frac{C_{Child,t}}{N_t^{X1}}\right)^{\tau} + \gamma_2 \cdot \left(\frac{E_t}{N_t^{X2}}\right)^{\tau} + \gamma_3 \cdot \left(\frac{L_{Husband,t}}{N_t^{X3}}\right)^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot \left(\frac{L_{Wife,t}}{N_t^{X4}}\right)^{\tau}\right]^{\frac{1}{\tau}}$$

Where Q_t is the quality of each child and is a function of the consumption expenditure per child $C_{Child,t}$, the education expenditure per child E_t, proportion of husband's labour time spent on child rearing $L_{Husband,t}$ $(0 < L_{Husband,t} < 1)$ in each period t, the proportion of wife's labour time spent on child rearing $L_{Wife,t}$ $(0 < L_{Wife,t} < 1)$ in each period t; $0 < \gamma_1 < 1$ represents the portion of the quality contributed by $C_{Child,t}$, $0 < \gamma_2 < 1$ represents the proportion of the quality contributed by E_t, $0 < \gamma_3 < 1$ represents the proportion of the quality contributed by $L_{Husband,t}$, $0 < 1 - \gamma_1 - \gamma_2 - \gamma_3 < 1$ represents the proportion of the quality contributed by $L_{Wife,t}$; $\frac{1}{\tau}$ with $-\infty < \tau < 1$ represents the elasticity of substitution among the factors of $C_{Child,t}$, E_t, $L_{Husband,t}$ and $L_{Wife,t}$ contributed to the quality of each child; χ_1, χ_2, χ_3 and χ_4 represent the economies of scale in the consumption expenditure, education expenditure, husband's labour time and wife's labour time spent on child rearing. Miller (2008) indicates that this type of constant elasticity of substitution function is popular among economist due to its flexibility regarding the degree of substitution between inputs.

3.4 Infertility Risk of Husband and Wife

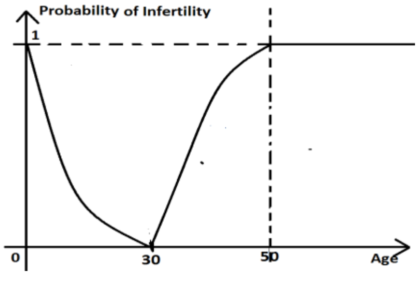
Enlightened by Sommer's (2012) research work of incorporating infertility risk in fertility choice models to build more realism into models, assume husband and wife face a binary idiosyncratic infertility shock $\mathrm{Infertility}_t$ which arrives at beginning of each period t, the behavior of $\mathrm{Infertility}_t$ is summarized in the following table.

Table 2: Behaviour of $\mathbf{Infertility_t}$

Age	Probability of Infertility
0 ≤t≤30	e^{-t}
30<t≤50	Ln(t-30)
t>50	1

The probability density of $\mathrm{Infertility}_t$ is based on the fertility cycle of wife, it reflects the reality that women's infertility risk reduces before the age of 30, infertility risk increases after the age of 30, and do not conceive babies after the age of 50. $\overline{H_{Husband}}$ is husband's general health level until age 50 in terms of $\overline{H_{Husband}} = \frac{\sum_{t=1}^{50} H_{Husband,t}}{50}$, $\overline{H_{Wife}}$ is wife's general health level until age 50 in terms of $\overline{H_{Wife}} = \frac{\sum_{t=1}^{50} H_{Wife,t}}{50}$. A_1 is a decreasing function of $\overline{H_{Husband}}$ and $\overline{H_{Wife}}$ since husband's and wife's infertility risk reduce as their general level of health increase.

Graph 1: Age and Probability of Infertility



According to the graph, the probability density function of $\mathrm{Infertility}_t$ is convex with respect to age before the age of 30, concave with respect to age between the age of 30 and 50, and becomes a constant of 1 after the age of 50, the inflection point where curve changes from convex to concave is set at age =30.

3.5 Household's Wealth Constraint

From the age of 18 to the age 65, husband earns wage of $W_{Husband,t}$ at each period t while wife earns wage of $W_{Wife,t}$ at each period t. Husband receives constant annual pension $\overline{\mathrm{Pension}_{Husband}}$ from the government when $t \geq 66$, while wife receives constant annual pension $\overline{\mathrm{Pension}_{\mathrm{Wife}}}$ from the government when $t \geq 66$. Each child enables the household to receive child rearing an annual subsidy of S_t from the government from birth until the age of 18. The household pays an annual education expense of E_t to each child from the child's birth until the child reaches the age of 18.

$$\sum_{t=18}^{65}\beta_{\mathrm{Husband}}^{t-18}\cdot\left[W_{\mathrm{Husband},t}\cdot\left(1-L_{\mathrm{Husband},t}\right)+W_{\mathrm{Wife},t}\cdot\left(1-L_{\mathrm{Wife},t}\right)+S_t\cdot N_t\right]+\sum_{t=66}^{100}\beta_{\mathrm{Husband}}^{t-18}\cdot\left(\overline{\mathrm{Pension}_{\mathrm{Husband}}}+\overline{\mathrm{Pension}_{\mathrm{Wife}}}+S_t\cdot N_t\right)=\sum_{t=18}^{100}\beta_{\mathrm{Husband}}^{t-18}\cdot\left[C_{\mathrm{Husband},t}+C_{\mathrm{Wife},t}+C_{\mathrm{Child},t}\cdot N_t+E_t\cdot N_t\right]$$

3.6 Household's Fertility Decision Making

3.6.1 Husband's Decision Making

$$\mathrm{Lagrangian}_{\mathrm{Husband}}\left(C_{\mathrm{Husband},t},L_{\mathrm{Husband},t},N_t,K_t\right)=\sum_{t=18}^{100}\beta_{\mathrm{Husband}}^{t-18}\cdot\left[\frac{C_{\mathrm{Husband},t}^{1-\alpha_{\mathrm{Husband}}}}{1-\alpha_{\mathrm{Husband}}}+\frac{(Q_t\cdot N_t)^{1-\alpha_{\mathrm{Husband}}}}{1-\alpha_{\mathrm{Husband}}}\right]+\lambda_{\mathrm{Husband}}\cdot\left\{\sum_{t=18}^{65}\beta_{\mathrm{Husband}}^{t-18}\cdot\left[W_{\mathrm{Husband},t}\cdot\left(1-L_{\mathrm{Husband},t}\right)+W_{\mathrm{Wife},t}\cdot\left(1-L_{\mathrm{Wife},t}\right)+S_t\cdot N_t\right]+\sum_{t=66}^{100}\beta_{\mathrm{Husband}}^{t-18}\cdot\left(\overline{\mathrm{Pension}_{\mathrm{Husband}}}+\overline{\mathrm{Pension}_{\mathrm{Wife}}}+S_t\cdot N_t\right)-\sum_{t=18}^{100}\beta_{\mathrm{Husband}}^{t-18}\cdot\left[C_{\mathrm{Husband},t}+C_{\mathrm{Wife},t}+C_{\mathrm{Child},t}\cdot N_t+E_t\cdot N_t\right]\right\}$$

$$=\sum_{t=18}^{100}\beta_{\mathrm{Husband}}^{t-18}\cdot\left\{\frac{C_{\mathrm{Husband},t}^{1-\alpha_{\mathrm{Husband}}}}{1-\alpha_{\mathrm{Husband}}}+\frac{\left\{\left[\gamma_1\cdot\left(\frac{C_{\mathrm{Child},t}}{N_t^{\chi_1}}\right)^{\tau}+\gamma_2\cdot\left(\frac{E_t}{N_t^{\chi_2}}\right)^{\tau}+\gamma_3\cdot\left(\frac{L_{\mathrm{Husband},t}}{N_t^{\chi_3}}\right)^{\tau}+(1-\gamma_1-\gamma_2-\gamma_3)\cdot\left(\frac{L_{\mathrm{Wife},t}}{N_t^{\chi_4}}\right)^{\tau}\right]^{\frac{1}{\tau}}\cdot N_t\right\}^{1-\alpha_{\mathrm{Husband}}}}{1-\alpha_{\mathrm{Husband}}}\right\}+\lambda_{\mathrm{Husband}}\cdot\left\{\sum_{t=18}^{65}\beta_{\mathrm{Husband}}^{t-18}\cdot\left[W_{\mathrm{Husband},t}\cdot\left(1-L_{\mathrm{Husband},t}\right)+W_{\mathrm{Wife},t}\cdot\left(1-L_{\mathrm{Wife},t}\right)+S_t\cdot N_t\right]+\sum_{t=66}^{100}\beta_{\mathrm{Husband}}^{t-18}\cdot\left(\overline{\mathrm{Pension}_{\mathrm{Husband}}}+\overline{\mathrm{Pension}_{\mathrm{Wife}}}+S_t\cdot N_t\right)-\sum_{t=18}^{100}\beta_{\mathrm{Husband}}^{t-18}\cdot\left[C_{\mathrm{Husband},t}+C_{\mathrm{Wife},t}+C_{\mathrm{Child},t}\cdot N_t+E_t\cdot N_t\right]\right\}$$

In order to obtain an analytic solution, assume $\chi_1=\chi_2=\chi_3=\chi_4=\chi$, the above equation becomes the following.

$$\sum_{t=18}^{100}\beta_{\mathrm{Husband}}^{t-18}\cdot\left\{\frac{C_{\mathrm{Husband},t}^{1-\alpha_{\mathrm{Husband}}}}{1-\alpha_{\mathrm{Husband}}}+\left\{\left[\gamma_1\cdot\left(\frac{C_{\mathrm{Child},t}}{N_t^{\chi}}\right)^{\tau}+\gamma_2\cdot\left(\frac{E_t}{N_t^{\chi}}\right)^{\tau}+\gamma_3\cdot\left(\frac{L_{\mathrm{Husband},t}}{N_t^{\chi}}\right)^{\tau}+(1-\gamma_1-\gamma_2-\gamma_3)\cdot\left(\frac{L_{\mathrm{Wife},t}}{N_t^{\chi}}\right)^{\tau}\right]^{\frac{1}{\tau}}\cdot N_t\right\}^{1-\alpha_{\mathrm{Husband}}}/(1-\alpha_{\mathrm{Husband}})\right\}+\lambda_{\mathrm{Husband}}\cdot\left\{\sum_{t=18}^{65}\beta_{\mathrm{Husband}}^{t-18}\cdot\left[W_{\mathrm{Husband},t}\cdot\left(1-L_{\mathrm{Husband},t}\right)+W_{\mathrm{Wife},t}\cdot\left(1-L_{\mathrm{Wife},t}\right)+S_t\cdot N_t\right]+\sum_{t=66}^{100}\beta_{\mathrm{Husband}}^{t-18}\cdot\left(\overline{\mathrm{Pension}_{\mathrm{Husband}}}+\overline{\mathrm{Pension}_{\mathrm{Wife}}}+S_t\cdot N_t\right)-\sum_{t=18}^{100}\beta_{\mathrm{Husband}}^{t-18}\cdot\left[C_{\mathrm{Husband},t}+C_{\mathrm{Wife},t}+C_{\mathrm{Child},t}\cdot N_t+E_t\cdot N_t\right]\right\}$$

$$=\sum_{t=18}^{100}\beta_{\mathrm{Husband}}^{t-18}\cdot\left\{\frac{C_{\mathrm{Husband},t}^{1-\alpha_{\mathrm{Husband}}}}{1-\alpha_{\mathrm{Husband}}}+\left\{\left[\gamma_1\cdot\left(\frac{C_{\mathrm{Child},t}}{\left(\sum_{j=18}^{t}K_j\cdot I_{t-j<18}\right)^{\chi}}\right)^{\tau}+\gamma_2\cdot\left(\frac{E_t}{\left(\sum_{j=18}^{t}K_j\cdot I_{t-j<18}\right)^{\chi}}\right)^{\tau}+\gamma_3\cdot\right.\right.\right.$$

$$\left(\frac{L_{Husband,t}}{\left(\sum_{j=18}^{t} K_j \cdot I_{t-j<18}\right)^{\chi}}\right)^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3)\cdot\left(\frac{L_{Wife,t}}{\left(\sum_{j=18}^{t} K_j \cdot I_{t-j<18}\right)^{\chi}}\right)^{\tau}\Bigg]^{\frac{1}{\tau}} \cdot \left(\sum_{j=18}^{t} K_j \cdot I_{t-j<18}\right)\Bigg\}^{1-\alpha_{Husband}} \Bigg/ (1-\alpha_{Husband})\Bigg\} + \lambda_{Husband}\cdot\Big\{\sum_{t=18}^{65}\beta_{Husband}^{t-18}\cdot\big[W_{Husband,t}\cdot(1-L_{Husband,t}) + W_{Wife,t}\cdot(1-L_{Wife,t}) + S_t\cdot\left(\sum_{j=18}^{t} K_j \cdot I_{t-j<18}\right)\big] + \sum_{t=66}^{100}\beta_{Husband}^{t-18}\cdot\left(\overline{Pension_{Husband}}+\overline{Pension_{Wife}} + S_t\cdot\left(\sum_{j=18}^{t} K_j \cdot I_{t-j<18}\right)\right) - \sum_{t=18}^{100}\beta_{Husband}^{t-18}\cdot\big[C_{Husband,t}+C_{Wife,t}+C_{Child,t}\cdot\left(\sum_{j=18}^{t} K_j \cdot I_{t-j<18}\right)+E_t\cdot\left(\sum_{j=18}^{t} K_j \cdot I_{t-j<18}\right)\big]\Big\}$$

$$= \sum_{t=18}^{100}\beta_{Husband}^{t-18}\cdot\Bigg\{\frac{C_{Husband,t}^{1-\alpha_{Husband}}}{1-\alpha_{Husband}} + \left[\gamma_1\cdot {C_{Child,t}}^{\tau} + \gamma_2\cdot {E_t}^{\tau} + \gamma_3\cdot {L_{Husband,t}}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3)\cdot {L_{Wife,t}}^{\tau}\right]^{\frac{(1-\alpha_{Husband})}{\tau}}\cdot\left(\sum_{j=18}^{t} K_j \cdot I_{t-j<18}\right)^{(1-\chi)\cdot(1-\alpha_{Husband})}\Big/(1-\alpha_{Husband})\Bigg\} + \lambda_{Husband}\cdot\Big\{\sum_{t=18}^{65}\beta_{Husband}^{t-18}\cdot\big[W_{Husband,t}\cdot(1-L_{Husband,t}) + W_{Wife,t}\cdot(1-L_{Wife,t}) + S_t\cdot\left(\sum_{j=18}^{t} K_j \cdot I_{t-j<18}\right)\big] + \sum_{t=66}^{100}\beta_{Husband}^{t-18}\cdot\left(\overline{Pension_{Husband}}+\overline{Pension_{Wife}} + S_t\cdot\left(\sum_{j=18}^{t} K_j \cdot I_{t-j<18}\right)\right) - \sum_{t=18}^{100}\beta_{Husband}^{t-18}\cdot\big[C_{Husband,t}+C_{Wife,t}+C_{Child,t}\cdot\left(\sum_{j=18}^{t} K_j \cdot I_{t-j<18}\right)+E_t\cdot\left(\sum_{j=18}^{t} K_j \cdot I_{t-j<18}\right)\big]\Big\}$$

Take the Lagrangian function with respect to $C_{Husband,t}$, $L_{Husband,t}$, N_t, K_t.

$$\frac{\partial Lagrangian_{Husband}}{\partial C_{Husband,t}} = \beta_{Husband}^{t-18}\cdot {C_{Husband,t}}^{-\alpha_{Husband}} - \lambda_{Husband}\cdot\beta_{Husband}^{t-18} = 0$$

$$\frac{\partial Lagrangian_{Husband}}{\partial L_{Husband,t}} = \beta_{Husband}^{t-18}\cdot\frac{1-\alpha_{Husband}}{\tau}\cdot\left(\gamma_1\cdot {C_{Child,t}}^{\tau} + \gamma_2\cdot {E_t}^{\tau} + \gamma_3\cdot {L_{Husband,t}}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3)\cdot {L_{Wife,t}}^{\tau}\right)^{\frac{1-\alpha_{Husband}-\tau}{\tau}}\cdot {N_t}^{(1-\chi)\cdot(1-\alpha_{Husband})}\cdot\gamma_3\cdot\tau\cdot {L_{Husband,t}}^{\tau-1} - \lambda_{Husband}\cdot\beta_{Husband}^{t-18}\cdot W_{Husband,t} = 0$$

$$\frac{\partial Lagrangian_{Husband}}{\partial N_t} = \beta_{Husband}^{t-18}\cdot\left[\gamma_1\cdot {C_{Child,t}}^{\tau} + \gamma_2\cdot {E_t}^{\tau} + \gamma_3\cdot {L_{Husband,t}}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3)\cdot {L_{Wife,t}}^{\tau}\right]^{\frac{1-\alpha_{Husband}}{\tau}}\cdot(1-\chi)\cdot {N_t}^{-\alpha_{Husband}-\chi+\chi\cdot\alpha_{Husband}} + \lambda_{Husband}\cdot\beta_{Husband}^{t-18}\cdot\left(S_t - C_{Child,t} - E_t\right) = 0$$

$$\frac{\partial Lagrangian_{Husband}}{\partial K_t} = \Bigg\{\beta_{Husband}^{t-18}\left[\gamma_1\cdot {C_{Child,t}}^{\tau} + \gamma_2\cdot {E_t}^{\tau} + \gamma_3\cdot {L_{Husband,t}}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3)\cdot {L_{Wife,t}}^{\tau}\right]^{\frac{1-\alpha_{Husband}}{\tau}} + \beta_{Husband}^{t-19}\left[\gamma_1\cdot {C_{Child,t+1}}^{\tau} + \gamma_2\cdot {E_{t+1}}^{\tau} + \gamma_3\cdot {L_{Husband,t+1}}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3)\cdot {L_{Wife,t+1}}^{\tau}\right]^{\frac{1-\alpha_{Husband}}{\tau}} + \cdots + \beta_{Husband}^{0}\left[\gamma_1\cdot {C_{Child,2t-18}}^{\tau} + \gamma_2\cdot {E_{2t-18}}^{\tau} + \gamma_3\cdot {L_{Husband,2t-18}}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3)\cdot {L_{Wife,2t-18}}^{\tau}\right]^{\frac{1-\alpha_{Husband}}{\tau}}\Bigg\}\cdot {K_t}^{-\alpha_{Husband}-\chi+\chi\cdot\alpha_{Husband}}\cdot(1-\chi) + \lambda_{Husband}\cdot\left[\beta_{Husband}^{t-18}\cdot\left(S_t - C_{Child,t} - E_t\right) + \beta_{Husband}^{t-19}\cdot\left(S_{t+1} - C_{Child,t+1} - E_{t+1}\right) + \cdots + \beta_{Husband}^{0}\cdot\left(S_{2t-18} - C_{Child,2t-18} - E_{2t-18}\right)\right] = 0$$

Combine $\frac{\partial Lagrangian_{Husband}}{\partial C_{Husband,t}} = 0$ and $\frac{\partial Lagrangian_{Husband}}{\partial L_{Husband,t}} = 0$ to obtain the following.

$\beta_{Husband}^{t-18} \cdot \frac{(1-\alpha_{Husband})}{\tau} \cdot$
$\left(\gamma_1 \cdot C_{Child,t}{}^{\tau} + \gamma_2 \cdot E_t{}^{\tau} + \gamma_3 \cdot L_{Husband,t}{}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot L_{Wife,t}{}^{\tau}\right)^{\frac{(1-\alpha_{Husband}-\tau)}{\tau}} \cdot$
$N_t{}^{(1-\chi)\cdot(1-\alpha_{Husband})} \cdot (1-\gamma_1-\gamma_2-\gamma_3) \cdot \tau \cdot L_{Husband,t}{}^{\tau-1} = C_{Husband,t}{}^{-\alpha_{Husband}} \cdot \beta_{Husband}^{t-18} \cdot$
$W_{Husband,t} \rightarrow$

$$N_t = W_{Husband,t}{}^{\frac{1}{(1-\chi)\cdot(1-\alpha_{Husband})}} \cdot L_{Husband,t}{}^{\frac{(1-\tau)}{(1-\chi)\cdot(1-\alpha_{Husband})}} \cdot \left(\gamma_1 \cdot C_{Child,t}{}^{\tau} + \gamma_2 \cdot E_t{}^{\tau} + \gamma_3 \cdot L_{Husband,t}{}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot L_{Wife,t}{}^{\tau}\right)^{\frac{1}{\tau}\cdot\frac{(\tau+\alpha_{Husband}-1)}{(1-\chi)\cdot(1-\alpha_{Husband})}} / \left[(1-\alpha_{Husband})^{\frac{1}{(1-\chi)\cdot(1-\alpha_{Husband})}} \cdot (1-\gamma_1-\gamma_2-\gamma_3)^{\frac{1}{(1-\chi)\cdot(1-\alpha_{Husband})}} \cdot C_{Husband,t}{}^{\frac{\alpha_{Husband}}{(1-\chi)\cdot(1-\alpha_{Husband})}}\right]$$

The number of children N_t that husband would like to have is positively related with husband's wage $W_{Husband,t}$, the proportion of husband's labour spent on child rearing $L_{Husband,t}$ and the quality of child $\left(\gamma_1 \cdot C_{Child,t}{}^{\tau} + \gamma_2 \cdot E_t{}^{\tau} + \gamma_3 \cdot L_{Husband,t}{}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot L_{Wife,t}{}^{\tau}\right)^{\frac{1}{\tau}}$; while negatively correlated with husband's consumption $C_{Husband,t}$.

Combine $\frac{\partial Lagrangian_{Husband}}{\partial L_{Husband,t}} = 0$ and $\frac{\partial Lagrangian_{Husband}}{\partial N_t} = 0$ to obtain the following.

$$\frac{1-\alpha_{Husband}}{\tau} \cdot \left(\gamma_1 \cdot C_{Child,t}{}^{\tau} + \gamma_2 \cdot E_t{}^{\tau} + \gamma_3 \cdot L_{Husband,t}{}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot L_{Wife,t}{}^{\tau}\right)^{\frac{1-\alpha_{Husband}-\tau}{\tau}} \cdot N_t{}^{(1-\chi)\cdot(1-\alpha_{Husband})} \cdot \gamma_3 \cdot \tau \cdot L_{Husband,t}{}^{\tau-1} / W_{Husband,t}$$
$$= \left[\gamma_1 \cdot C_{Child,t}{}^{\tau} + \gamma_2 \cdot E_t{}^{\tau} + \gamma_3 \cdot L_{Husband,t}{}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot L_{Wife,t}{}^{\tau}\right]^{\frac{1-\alpha_{Husband}}{\tau}} \cdot (1-\chi) \cdot N_t{}^{-\alpha_{Husband}-\chi+\chi\cdot\alpha_{Husband}} / \left(C_{Child,t} + E_t - S_t\right)$$

$$\rightarrow N_t = \left[\gamma_1 \cdot C_{Child,t}{}^{\tau} + \gamma_2 \cdot E_t{}^{\tau} + \gamma_3 \cdot L_{Husband,t}{}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot L_{Wife,t}{}^{\tau}\right]^{\frac{1-\alpha_{Husband}}{\tau}} \cdot (1-\chi) \cdot N_t{}^{-\alpha_{Husband}-\chi+\chi\cdot\alpha_{Husband}} \cdot W_{Husband,t} / \left\{\left(C_{Child,t} + E_t - S_t\right) \cdot \frac{1-\alpha_{Husband}}{\tau} \cdot \left(\gamma_1 \cdot C_{Child,t}{}^{\tau} + \gamma_2 \cdot E_t{}^{\tau} + \gamma_3 \cdot L_{Husband,t}{}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot L_{Wife,t}{}^{\tau}\right)^{\frac{1-\alpha_{Husband}-\tau}{\tau}} \cdot \gamma_3 \cdot \tau \cdot L_{Husband,t}{}^{\tau-1}\right\}$$

$$\rightarrow N_t = \left[\gamma_1 \cdot C_{Child,t}{}^{\tau} + \gamma_2 \cdot E_t{}^{\tau} + \gamma_3 \cdot L_{Husband,t}{}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot L_{Wife,t}{}^{\tau}\right]^{\frac{1}{\tau}\cdot\frac{\tau}{(1+\alpha_{Husband}+\chi-\chi\cdot\alpha_{Husband})}} \cdot (1-\chi)^{\frac{1}{(1+\alpha_{Husband}+\chi-\chi\cdot\alpha_{Husband})}} \cdot W_{Husband,t}{}^{\frac{1}{(1+\alpha_{Husband}+\chi-\chi\cdot\alpha_{Husband})}} \cdot L_{Husband,t}{}^{\frac{(1-\tau)}{(1+\alpha_{Husband}+\chi-\chi\cdot\alpha_{Husband})}} / \left[\left(C_{Child,t} + E_t - S_t\right) \cdot (1-\alpha_{Husband}) \cdot \gamma_3\right]^{\frac{1}{(1+\alpha_{Husband}+\chi-\chi\cdot\alpha_{Husband})}}$$

The number of children N_t that husband would like to have is positively related with the quality of child $\left(\gamma_1 \cdot C_{Child,t}{}^{\tau} + \gamma_2 \cdot E_t{}^{\tau} + \gamma_3 \cdot L_{Husband,t}{}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot L_{Wife,t}{}^{\tau}\right)^{\frac{1}{\tau}}$ and the child's subsidy S_t; while negatively correlated with husband's consumption $C_{Husband,t}$, the child's consumption $C_{Child,t}$ and the child's education expenses $E_{Child,t}$.

Combine $\frac{\partial Lagrangian_{Husband}}{\partial C_{Husband,t}} = 0$ and $\frac{\partial Lagrangian_{Husband}}{\partial K_t} = 0$,

$$\Big\{\beta_{Husband}^{t-18}\left[\gamma_1 \cdot C_{Child,t}{}^{\tau} + \gamma_2 \cdot E_t{}^{\tau} + \gamma_3 \cdot L_{Husband,t}{}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot L_{Wife,t}{}^{\tau}\right]^{\frac{1-\alpha_{Husband}}{\tau}} + \beta_{Husband}^{t-19}\left[\gamma_1 \cdot C_{Child,t+1}{}^{\tau} + \gamma_2 \cdot E_{t+1}{}^{\tau} + \gamma_3 \cdot L_{Husband,t+1}{}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot L_{Wife,t+1}{}^{\tau}\right]^{\frac{1-\alpha_{Husband}}{\tau}} + \cdots + \beta_{Husband}^{0}\left[\gamma_1 \cdot C_{Child,2t-18}{}^{\tau} + \gamma_2 \cdot E_{2t-18}{}^{\tau} + \gamma_3 \cdot L_{Husband,2t-18}{}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot L_{Wife,2t-18}{}^{\tau}\right]^{\frac{1-\alpha_{Husband}}{\tau}}\Big\} \cdot K_t{}^{-\alpha_{Husband}-\chi+\chi\cdot\alpha_{Husband}} \cdot (1-\chi) = -C_{Husband,t}{}^{-\alpha_{Husband}} \cdot \left[\beta_{Husband}^{t-18} \cdot \left(S_t - C_{Child,t} - E_t\right) + \beta_{Husband}^{t-19} \cdot \left(S_{t+1} - C_{Child,t+1} - E_{t+1}\right) + \cdots + \beta_{Husband}^{0} \cdot \left(S_{2t-18} - C_{Child,2t-18} - E_{2t-18}\right)\right] \rightarrow$$

$$K_t = \Big\{\beta_{Husband}^{t-18}\left[\gamma_1 \cdot C_{Child,t}{}^{\tau} + \gamma_2 \cdot E_t{}^{\tau} + \gamma_3 \cdot L_{Husband,t}{}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot L_{Wife,t}{}^{\tau}\right]^{\frac{1-\alpha_{Husband}}{\tau}} + \beta_{Husband}^{t-19}\left[\gamma_1 \cdot C_{Child,t+1}{}^{\tau} + \gamma_2 \cdot E_{t+1}{}^{\tau} + \gamma_3 \cdot L_{Husband,t+1}{}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot L_{Wife,t+1}{}^{\tau}\right]^{\frac{1-\alpha_{Husband}}{\tau}} + \cdots + \beta_{Husband}^{0}\left[\gamma_1 \cdot C_{Child,2t-18}{}^{\tau} + \gamma_2 \cdot E_{2t-18}{}^{\tau} + \gamma_3 \cdot L_{Husband,2t-18}{}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot L_{Wife,2t-18}{}^{\tau}\right]^{\frac{1-\alpha_{Husband}}{\tau}}\Big\}^{\frac{1}{(\alpha_{Husband}+\chi-\chi\cdot\alpha_{Husband})}} \cdot C_{Husband,t}{}^{\frac{\alpha_{Husband}}{(\alpha_{Husband}+\chi-\chi\cdot\alpha_{Husband})}} / \left[\beta_{Husband}^{t-18} \cdot \left(C_{Child,t} + E_t - S_t\right) + \beta_{Husband}^{t-19} \cdot \left(C_{Child,t+1} + E_{t+1} - S_{t+1}\right) + \cdots + \beta_{Husband}^{0} \cdot \left(C_{Child,2t-18} + E_{2t-18} - S_{2t-18}\right)\right]^{\frac{1}{(\alpha_{Husband}+\chi-\chi\cdot\alpha_{Husband})}}$$

The probability of having a child in the current period $P(K_t = 1)$ is positively correlated with the qualities of all the current and previous children $\{Q_j\}_{j=t}^{j=2t-18}$, husband's consumption $C_{Husband,t}$, all the current and previous child subsidies $\{S_j\}_{j=t}^{j=2t-18}$; while negatively correlated with all the current and previous child consumption $\{C_j\}_{j=t}^{j=2t-18}$ and child education expenses $\{E_j\}_{j=t}^{j=2t-18}$.

The optimal number of children N_t is independent of husband's discount factor $\beta_{Husband}$, however, the probability of having a child in the current period $P(K_t = 1)$ is positively correlated with husband's discount factor $\beta_{Husband}$. Higher discount factor in terms of higher value of $\beta_{Husband}$ and lower risk aversion in terms of lower value of $\alpha_{Husband}$ increases the probability of having a child in the current period $P(K_t = 1)$.

3.6.2 Wife's Decision Making

$$Lagrangian_{Wife}\left(C_{Wife,t}, L_{Wife,t}, N_t, K_t\right) = \sum_{t=18}^{100} \beta_{Wife}^{t-18} \cdot \left[\frac{C_{Wife,t}^{1-\alpha_{Wife}}}{1-\alpha_{Wife}} + \frac{(Q_t \cdot N_t)^{1-\alpha_{Wife}}}{1-\alpha_{Wife}}\right] + \lambda_{Wife} \cdot \Big\{\sum_{t=18}^{65} \beta_{Wife}^{t-18} \cdot \left[W_{Husband,t} \cdot \left(1 - L_{Husband,t}\right) + W_{Wife,t} \cdot \left(1 - L_{Wife,t}\right) + S_t \cdot N_t\right] + \sum_{t=66}^{100} \beta_{Wife}^{t-18} \cdot \left(\overline{Pension_{Husband}} + \overline{Pension_{Wife}} + S_t \cdot N_t\right) - \sum_{t=18}^{100} \beta_{Wife}^{t-18} \cdot \left[C_{Husband,t} + C_{Wife,t} + C_{Child,t} \cdot N_t + E_t \cdot N_t\right]\Big\}$$

$$= \sum_{t=18}^{100} \beta_{\mathrm{Wife}}^{t-18} \cdot \left\{ \frac{C_{\mathrm{Wife},t}^{1-\alpha_{\mathrm{Wife}}}}{1-\alpha_{\mathrm{Wife}}} + \frac{\left\{ \left[\gamma_1 \cdot \left(\frac{C_{\mathrm{Child},t}}{N_t^{\chi_1}} \right)^{\tau} + \gamma_2 \cdot \left(\frac{E_t}{N_t^{\chi_2}} \right)^{\tau} + \gamma_3 \cdot \left(\frac{L_{\mathrm{Husband},t}}{N_t^{\chi_3}} \right)^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot \left(\frac{L_{\mathrm{Wife},t}}{N_t^{\chi_4}} \right)^{\tau} \right]^{\frac{1}{\tau}} \cdot N_t \right\}^{1-\alpha_{\mathrm{Wife}}}}{1-\alpha_{\mathrm{Wife}}} \right\} +$$

$$\lambda_{\mathrm{Wife}} \cdot \left\{ \sum_{t=18}^{65} \beta_{\mathrm{Wife}}^{t-18} \cdot \left[W_{\mathrm{Husband},t} \cdot \left(1 - L_{\mathrm{Husband},t}\right) + W_{\mathrm{Wife},t} \cdot \left(1 - L_{\mathrm{Wife},t}\right) + S_t \cdot N_t \right] + \sum_{t=66}^{100} \beta_{\mathrm{Husband}}^{t-18} \cdot \left(\overline{\mathrm{Pension}_{\mathrm{Husband}}} + \overline{\mathrm{Pension}_{\mathrm{Wife}}} + S_t \cdot N_t \right) - \sum_{t=18}^{100} \beta_{\mathrm{Wife}}^{t-18} \cdot \left[C_{\mathrm{Husband},t} + C_{\mathrm{Wife},t} + C_{\mathrm{Child},t} \cdot N_t + E_t \cdot N_t \right] \right\}$$

In order to obtain an analytic solution, assume $\chi_1 = \chi_2 = \chi_3 = \chi_4 = \chi$, the above equation becomes the following.

$$\sum_{t=18}^{100} \beta_{\mathrm{Wife}}^{t-18} \cdot \left\{ \frac{C_{\mathrm{Wife},t}^{1-\alpha_{\mathrm{Wife}}}}{1-\alpha_{\mathrm{Wife}}} + \left\{ \left[\gamma_1 \cdot \left(\frac{C_{\mathrm{Child},t}}{N_t^{\chi}} \right)^{\tau} + \gamma_2 \cdot \left(\frac{E_t}{N_t^{\chi}} \right)^{\tau} + \gamma_3 \cdot \left(\frac{L_{\mathrm{Husband},t}}{N_t^{\chi}} \right)^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot \left(\frac{L_{\mathrm{Wife},t}}{N_t^{\chi}} \right)^{\tau} \right]^{\frac{1}{\tau}} \cdot N_t \right\}^{1-\alpha_{\mathrm{Wife}}} / (1-\alpha_{\mathrm{Wife}}) \right\} + \lambda_{\mathrm{Wife}} \cdot \left\{ \sum_{t=18}^{65} \beta_{\mathrm{Wife}}^{t-18} \cdot \left[W_{\mathrm{Husband},t} \cdot \left(1 - L_{\mathrm{Husband},t}\right) + W_{\mathrm{Wife},t} \cdot \left(1 - L_{\mathrm{Wife},t}\right) + S_t \cdot N_t \right] + \sum_{t=66}^{100} \beta_{\mathrm{Husband}}^{t-18} \cdot \left(\overline{\mathrm{Pension}_{\mathrm{Husband}}} + \overline{\mathrm{Pension}_{\mathrm{Wife}}} + S_t \cdot N_t \right) - \sum_{t=18}^{100} \beta_{\mathrm{Wife}}^{t-18} \cdot \left[C_{\mathrm{Husband},t} + C_{\mathrm{Wife},t} + C_{\mathrm{Child},t} \cdot N_t + E_t \cdot N_t \right] \right\}$$

$$= \sum_{t=18}^{100} \beta_{\mathrm{Wife}}^{t-18} \cdot \left\{ \frac{C_{\mathrm{Wife},t}^{1-\alpha_{\mathrm{Wife}}}}{1-\alpha_{\mathrm{Wife}}} + \left\{ \left[\gamma_1 \cdot \left(\frac{C_{\mathrm{Child},t}}{\left(\sum_{j=18}^{t} K_j \cdot I_{t-j<18}\right)^{\chi}} \right)^{\tau} + \gamma_2 \cdot \left(\frac{E_t}{\left(\sum_{j=18}^{t} K_j \cdot I_{t-j<18}\right)^{\chi}} \right)^{\tau} + \gamma_3 \cdot \left(\frac{L_{\mathrm{Husband},t}}{\left(\sum_{j=18}^{t} K_j \cdot I_{t-j<18}\right)^{\chi}} \right)^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot \left(\frac{L_{\mathrm{Wife},t}}{\left(\sum_{j=18}^{t} K_j \cdot I_{t-j<18}\right)^{\chi}} \right)^{\tau} \right]^{\frac{1}{\tau}} \cdot \left(\sum_{j=18}^{t} K_j \cdot I_{t-j<18}\right) \right\}^{1-\alpha_{\mathrm{Wife}}} / (1-\alpha_{\mathrm{Wife}}) \right\} + \lambda_{\mathrm{Wife}} \cdot \left\{ \sum_{t=18}^{65} \beta_{\mathrm{Wife}}^{t-18} \cdot \left[W_{\mathrm{Husband},t} \cdot \left(1 - L_{\mathrm{Husband},t}\right) + W_{\mathrm{Wife},t} \cdot \left(1 - L_{\mathrm{Wife},t}\right) + S_t \cdot \left(\sum_{j=18}^{t} K_j \cdot I_{t-j<18}\right) \right] + \sum_{t=66}^{100} \beta_{\mathrm{Wife}}^{t-18} \cdot \left(\overline{\mathrm{Pension}_{\mathrm{Husband}}} + \overline{\mathrm{Pension}_{\mathrm{Wife}}} + S_t \cdot \left(\sum_{j=18}^{t} K_j \cdot I_{t-j<18}\right) \right) - \sum_{t=18}^{100} \beta_{\mathrm{Wife}}^{t-18} \cdot \left[C_{\mathrm{Husband},t} + C_{\mathrm{Wife},t} + C_{\mathrm{Child},t} \cdot \left(\sum_{j=18}^{t} K_j \cdot I_{t-j<18}\right) + E_t \cdot \left(\sum_{j=18}^{t} K_j \cdot I_{t-j<18}\right) \right] \right\}$$

$$= \sum_{t=18}^{100} \beta_{\mathrm{Wife}}^{t-18} \cdot \left\{ \frac{C_{\mathrm{Wife},t}^{1-\alpha_{\mathrm{Wife}}}}{1-\alpha_{\mathrm{Wife}}} + \left[\gamma_1 \cdot {C_{\mathrm{Child},t}}^{\tau} + \gamma_2 \cdot {E_t}^{\tau} + \gamma_3 \cdot {L_{\mathrm{Husband},t}}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot {L_{\mathrm{Wife},t}}^{\tau} \right]^{\frac{(1-\alpha_{\mathrm{Wife}})}{\tau}} \cdot \left(\sum_{j=18}^{t} K_j \cdot I_{t-j<18}\right)^{(1-\chi)\cdot(1-\alpha_{\mathrm{Wife}})} / (1-\alpha_{\mathrm{Wife}}) \right\} + \lambda_{\mathrm{Wife}} \cdot \left\{ \sum_{t=18}^{65} \beta_{\mathrm{Wife}}^{t-18} \cdot \left[W_{\mathrm{Husband},t} \cdot \left(1 - L_{\mathrm{Husband},t}\right) + W_{\mathrm{Wife},t} \cdot \left(1 - L_{\mathrm{Wife},t}\right) + S_t \cdot \left(\sum_{j=18}^{t} K_j \cdot I_{t-j<18}\right) \right] + \sum_{t=66}^{100} \beta_{\mathrm{Wife}}^{t-18} \cdot \left(\overline{\mathrm{Pension}_{\mathrm{Husband}}} + \overline{\mathrm{Pension}_{\mathrm{Wife}}} + S_t \cdot \left(\sum_{j=18}^{t} K_j \cdot I_{t-j<18}\right) \right) - \sum_{t=18}^{100} \beta_{\mathrm{Wife}}^{t-18} \cdot \left[C_{\mathrm{Husband},t} + C_{\mathrm{Wife},t} + C_{\mathrm{Child},t} \cdot \left(\sum_{j=18}^{t} K_j \cdot I_{t-j<18}\right) + E_t \cdot \left(\sum_{j=18}^{t} K_j \cdot I_{t-j<18}\right) \right] \right\}$$

Take the Lagrangian function with respect to $C_{\mathrm{Wife},t}$, $L_{\mathrm{Wife},t}$, N_t, K_t.

$$\frac{\partial \mathrm{Lagrangian}_{\mathrm{Wife}}}{\partial C_{\mathrm{Wife},t}} = \beta_{\mathrm{Wife}}^{t-18} \cdot {C_{\mathrm{Wife},t}}^{-\alpha_{\mathrm{Wife}}} - \lambda_{\mathrm{Wife}} \cdot \beta_{\mathrm{Wife}}^{t-18} = 0$$

$$\frac{\partial \mathrm{Lagrangian}_{\mathrm{Wife}}}{\partial L_{\mathrm{Wife},t}} = \beta_{\mathrm{Wife}}^{t-18} \cdot \frac{1-\alpha_{\mathrm{Wife}}}{\tau} \cdot \left(\gamma_1 \cdot {C_{\mathrm{Child},t}}^{\tau} + \gamma_2 \cdot {E_t}^{\tau} + \gamma_3 \cdot {L_{\mathrm{Husband},t}}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot {L_{\mathrm{Wife},t}}^{\tau} \right)^{\frac{1-\alpha_{\mathrm{Wife}}-\tau}{\tau}} \cdot {N_t}^{(1-\chi)\cdot(1-\alpha_{\mathrm{Wife}})} \cdot (1-\gamma_1-\gamma_2-\gamma_3) \cdot \tau \cdot {L_{\mathrm{Wife},t}}^{\tau-1} - \lambda_{\mathrm{Wife}} \cdot \beta_{\mathrm{Wife}}^{t-18} \cdot W_{\mathrm{Wife},t} = 0$$

$\frac{\partial \mathrm{Lagrangian}_{\mathrm{Wife}}}{\partial \mathrm{N_t}} =$
$\beta_{\mathrm{Wife}}^{\mathrm{t-18}} \cdot \left[\gamma_1 \cdot \mathrm{C_{Child,t}}^{\tau} + \gamma_2 \cdot \mathrm{E_t}^{\tau} + \gamma_3 \cdot \mathrm{L_{Husband,t}}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot \mathrm{L_{Wife,t}}^{\tau}\right]^{\frac{1-\alpha_{\mathrm{Wife}}}{\tau}} \cdot (1-\chi) \cdot \mathrm{N_t}^{-\alpha_{\mathrm{Wife}}-\chi+\chi\cdot\alpha_{\mathrm{Wife}}} + \lambda_{\mathrm{Wife}} \cdot \beta_{\mathrm{Wife}}^{\mathrm{t-18}} \cdot \left(\mathrm{S_t} - \mathrm{C_{Child,t}} - \mathrm{E_t}\right) = 0$

$\frac{\partial \mathrm{Lagrangian}_{\mathrm{Wife}}}{\partial \mathrm{K_t}} = \Big\{\beta_{\mathrm{Wife}}^{\mathrm{t-18}} \cdot \left[\gamma_1 \cdot \mathrm{C_{Child,t}}^{\tau} + \gamma_2 \cdot \mathrm{E_t}^{\tau} + \gamma_3 \cdot \mathrm{L_{Husband,t}}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot \mathrm{L_{Wife,t}}^{\tau}\right]^{\frac{1-\alpha_{\mathrm{Wife}}}{\tau}} + \beta_{\mathrm{Wife}}^{\mathrm{t-19}}\left[\gamma_1 \cdot \mathrm{C_{Child,t+1}}^{\tau} + \gamma_2 \cdot \mathrm{E_{t+1}}^{\tau} + \gamma_3 \cdot \mathrm{L_{Husband,t+1}}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot \mathrm{L_{Wife,t+1}}^{\tau}\right]^{\frac{1-\alpha_{\mathrm{Wife}}}{\tau}} + \cdots + \beta_{\mathrm{Wife}}^{0}\left[\gamma_1 \cdot \mathrm{C_{Child,2t-18}}^{\tau} + \gamma_2 \cdot \mathrm{E_{2t-18}}^{\tau} + \gamma_3 \cdot \mathrm{L_{Husband,2t-18}}^{\tau} + (1-\gamma_1 - \gamma_2-\gamma_3) \cdot \mathrm{L_{Wife,2t-18}}^{\tau}\right]^{\frac{1-\alpha_{\mathrm{Wife}}}{\tau}}\Big\} \cdot \mathrm{K_t}^{-\alpha_{\mathrm{Wife}}-\chi+\chi\cdot\alpha_{\mathrm{Wife}}} \cdot (1-\chi) + \lambda_{\mathrm{Wife}} \cdot \left[\beta_{\mathrm{Wife}}^{\mathrm{t-18}} \cdot \left(\mathrm{S_t} - \mathrm{C_{Child,t}} - \mathrm{E_t}\right) + \beta_{\mathrm{Wife}}^{\mathrm{t-19}} \cdot \left(\mathrm{S_{t+1}} - \mathrm{C_{Child,t+1}} - \mathrm{E_{t+1}}\right) + \cdots + \beta_{\mathrm{Wife}}^{0} \cdot \left(\mathrm{S_{2t-18}} - \mathrm{C_{Child,2t-18}} - \mathrm{E_{2t-18}}\right)\right] = 0$

Combine $\frac{\partial \mathrm{Lagrangian}_{\mathrm{Wife}}}{\partial \mathrm{C_{Wife,t}}} = 0$ and $\frac{\partial \mathrm{Lagrangian}_{\mathrm{Wife}}}{\partial \mathrm{L_{Wife,t}}} = 0$ to obtain the following.

$\beta_{\mathrm{Wife}}^{\mathrm{t-18}} \cdot \frac{(1-\alpha_{\mathrm{Wife}})}{\tau} \cdot \left(\gamma_1 \cdot \mathrm{C_{Child,t}}^{\tau} + \gamma_2 \cdot \mathrm{E_t}^{\tau} + \gamma_3 \cdot \mathrm{L_{Husband,t}}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot \mathrm{L_{Wife,t}}^{\tau}\right)^{\frac{(1-\alpha_{\mathrm{Wife}}-\tau)}{\tau}} \cdot \mathrm{N_t}^{(1-\chi)\cdot(1-\alpha_{\mathrm{Wife}})} \cdot (1-\gamma_1-\gamma_2-\gamma_3) \cdot \tau \cdot \mathrm{L_{Wife,t}}^{\tau-1} = \mathrm{C_{Wife,t}}^{-\alpha_{\mathrm{Wife}}} \cdot \beta_{\mathrm{Wife}}^{\mathrm{t-18}} \cdot \mathrm{W_{Wife,t}} \rightarrow$
$\mathrm{N_t} = \mathrm{W_{Wife,t}}^{\frac{1}{(1-\chi)\cdot(1-\alpha_{\mathrm{Wife}})}} \cdot \mathrm{L_{Wife,t}}^{\frac{(1-\tau)}{(1-\chi)\cdot(1-\alpha_{\mathrm{Wife}})}} \cdot \left(\gamma_1 \cdot \mathrm{C_{Child,t}}^{\tau} + \gamma_2 \cdot \mathrm{E_t}^{\tau} + \gamma_3 \cdot \mathrm{L_{Husband,t}}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot \mathrm{L_{Wife,t}}^{\tau}\right)^{\frac{1}{\tau}\cdot\frac{(\tau+\alpha_{\mathrm{Wife}}-1)}{(1-\chi)\cdot(1-\alpha_{\mathrm{Wife}})}} / \left[(1-\alpha_{\mathrm{Wife}})^{\frac{1}{(1-\chi)\cdot(1-\alpha_{\mathrm{Wife}})}} \cdot (1-\gamma_1-\gamma_2-\gamma_3)^{\frac{1}{(1-\chi)\cdot(1-\alpha_{\mathrm{Wife}})}} \cdot \mathrm{C_{Wife,t}}^{\frac{\alpha_{\mathrm{Wife}}}{(1-\chi)\cdot(1-\alpha_{\mathrm{Wife}})}}\right]$

The number of children $\mathrm{N_t}$ that wife would like to have is positively related with wife's wage $\mathrm{W_{Wife,t}}$, the proportion of wife's labour spent on child rearing $\mathrm{L_{Wife,t}}$ and the quality of child $\left(\gamma_1 \cdot \mathrm{C_{Child,t}}^{\tau} + \gamma_2 \cdot \mathrm{E_t}^{\tau} + \gamma_3 \cdot \mathrm{L_{Husband,t}}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot \mathrm{L_{Wife,t}}^{\tau}\right)^{\frac{1}{\tau}}$; while negatively correlated with wife's consumption $\mathrm{C_{Wife,t}}$.

Combine $\frac{\partial \mathrm{Lagrangian}_{\mathrm{Wife}}}{\partial \mathrm{L_{Wife,t}}} = 0$ and $\frac{\partial \mathrm{Lagrangian}_{\mathrm{Wife}}}{\partial \mathrm{N_t}} = 0$ to obtain the following.

$\Big\{\beta_{\mathrm{Wife}}^{\mathrm{t-18}}\left[\gamma_1 \cdot \mathrm{C_{Child,t}}^{\tau} + \gamma_2 \cdot \mathrm{E_t}^{\tau} + \gamma_3 \cdot \mathrm{L_{Husband,t}}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot \mathrm{L_{Wife,t}}^{\tau}\right]^{\frac{1-\alpha_{\mathrm{Wife}}}{\tau}} + \beta_{\mathrm{Wife}}^{\mathrm{t-19}}\left[\gamma_1 \cdot \mathrm{C_{Child,t+1}}^{\tau} + \gamma_2 \cdot \mathrm{E_{t+1}}^{\tau} + \gamma_3 \cdot \mathrm{L_{Husband,t+1}}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot \mathrm{L_{Wife,t+1}}^{\tau}\right]^{\frac{1-\alpha_{\mathrm{Wife}}}{\tau}} + \cdots + \beta_{\mathrm{Wife}}^{0}\left[\gamma_1 \cdot \mathrm{C_{Child,2t-18}}^{\tau} + \gamma_2 \cdot \mathrm{E_{2t-18}}^{\tau} + \gamma_3 \cdot \mathrm{L_{Husband,2t-18}}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot \mathrm{L_{Wife,2t-18}}^{\tau}\right]^{\frac{1-\alpha_{\mathrm{Wife}}}{\tau}}\Big\} \cdot \mathrm{K_t}^{-\alpha_{\mathrm{Wife}}-\chi+\chi\cdot\alpha_{\mathrm{Wife}}} \cdot (1-\chi) = -\mathrm{C_{Wife,t}}^{-\alpha_{\mathrm{Wife}}} \cdot \left[\beta_{\mathrm{Wife}}^{\mathrm{t-18}} \cdot \left(\mathrm{S_t} - \mathrm{C_{Child,t}} - \mathrm{E_t}\right) + \beta_{\mathrm{Wife}}^{\mathrm{t-19}} \cdot \left(\mathrm{S_{t+1}} - \mathrm{C_{Child,t+1}} - \mathrm{E_{t+1}}\right) + \cdots + \beta_{\mathrm{Wife}}^{0} \cdot \left(\mathrm{S_{2t-18}} - \mathrm{C_{Child,2t-18}} - \mathrm{E_{2t-18}}\right)\right] \rightarrow$

$$K_t = \Big\{\beta_{Wife}^{t-18}[\gamma_1 \cdot C_{Child,t}{}^{\tau} + \gamma_2 \cdot E_t{}^{\tau} + \gamma_3 \cdot L_{Husband,t}{}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot L_{Wife,t}{}^{\tau}]^{\frac{1-\alpha_{Wife}}{\tau}} + \beta_{Wife}^{t-19}[\gamma_1 \cdot C_{Child,t+1}{}^{\tau} + \gamma_2 \cdot E_{t+1}{}^{\tau} + \gamma_3 \cdot L_{Husband,t+1}{}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot L_{Wife,t+1}{}^{\tau}]^{\frac{1-\alpha_{Wife}}{\tau}} + \cdots + \beta_{Wife}^{0}[\gamma_1 \cdot C_{Child,2t-18}{}^{\tau} + \gamma_2 \cdot E_{2t-18}{}^{\tau} + \gamma_3 \cdot L_{Husband,2t-18}{}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot L_{Wife,2t-18}{}^{\tau}]^{\frac{1-\alpha_{Wife}}{\tau}}\Big\}^{\frac{1}{(\alpha_{Wife}+\chi-\chi\cdot\alpha_{Wife})}} \cdot C_{Wife,t}{}^{\frac{\alpha_{Wife}}{(\alpha_{Wife}+\chi-\chi\cdot\alpha_{Wife})}} / [\beta_{Wife}^{t-18} \cdot (C_{Child,t} + E_t - S_t) + \beta_{Wife}^{t-19} \cdot (C_{Child,t+1} + E_{t+1} - S_{t+1}) + \cdots + \beta_{Husband}^{0} \cdot (C_{Child,2t-18} + E_{2t-18} - S_{2t-18})]^{\frac{1}{(\alpha_{Wife}+\chi-\chi\cdot\alpha_{Wife})}}$$

The number of children N_t that wife would like to have is positively related with the quality of child $(\gamma_1 \cdot C_{Child,t}{}^{\tau} + \gamma_2 \cdot E_t{}^{\tau} + \gamma_3 \cdot L_{Husband,t}{}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot L_{Wife,t}{}^{\tau})^{\frac{1}{\tau}}$ and the child's subsidy S_t; while negatively correlated with wife's consumption $C_{Wife,t}$, the child's consumption $C_{Child,t}$ and the child's education expenses $E_{Child,t}$.

Combine $\frac{\partial Lagrangian_{Wife}}{\partial C_{Wife,t}} = 0$ and $\frac{\partial Lagrangian_{Wife}}{\partial K_t} = 0$ to obtain the following.

$$\Big\{\beta_{Wife}^{t-18} \cdot [\gamma_1 \cdot C_{Child,t}{}^{\tau} + \gamma_2 \cdot E_t{}^{\tau} + \gamma_3 \cdot L_{Husband,t}{}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot L_{Wife,t}{}^{\tau}]^{\frac{1-\alpha_{Wife}}{\tau}} + \beta_{Wife}^{t-19}[\gamma_1 \cdot C_{Child,t+1}{}^{\tau} + \gamma_2 \cdot E_{t+1}{}^{\tau} + \gamma_3 \cdot L_{Husband,t+1}{}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot L_{Wife,t+1}{}^{\tau}]^{\frac{1-\alpha_{Wife}}{\tau}} + \cdots + \beta_{Wife}^{0}[\gamma_1 \cdot C_{Child,2t-18}{}^{\tau} + \gamma_2 \cdot E_{2t-18}{}^{\tau} + \gamma_3 \cdot L_{Husband,2t-18}{}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot L_{Wife,2t-18}{}^{\tau}]^{\frac{1-\alpha_{Wife}}{\tau}}\Big\} \cdot K_t{}^{-\alpha_{Wife}-\chi+\chi\cdot\alpha_{Wife}} \cdot (1-\chi) = -C_{Wife,t}{}^{-\alpha_{Wife}} \cdot [\beta_{Wife}^{t-18} \cdot (S_t - C_{Child,t} - E_t) + \beta_{Wife}^{t-19} \cdot (S_{t+1} - C_{Child,t+1} - E_{t+1}) + \cdots + \beta_{Wife}^{0} \cdot (S_{2t-18} - C_{Child,2t-18} - E_{2t-18})] \rightarrow$$

$$K_t = \Big\{\beta_{Wife}^{t-18} \cdot [\gamma_1 \cdot C_{Child,t}{}^{\tau} + \gamma_2 \cdot E_t{}^{\tau} + \gamma_3 \cdot L_{Husband,t}{}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot L_{Wife,t}{}^{\tau}]^{\frac{1-\alpha_{Wife}}{\tau}} + \beta_{Wife}^{t-19}[\gamma_1 \cdot C_{Child,t+1}{}^{\tau} + \gamma_2 \cdot E_{t+1}{}^{\tau} + \gamma_3 \cdot L_{Husband,t+1}{}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot L_{Wife,t+1}{}^{\tau}]^{\frac{1-\alpha_{Wife}}{\tau}} + \cdots + \beta_{Wife}^{0}[\gamma_1 \cdot C_{Child,2t-18}{}^{\tau} + \gamma_2 \cdot E_{2t-18}{}^{\tau} + \gamma_3 \cdot L_{Husband,2t-18}{}^{\tau} + (1-\gamma_1-\gamma_2-\gamma_3) \cdot L_{Wife,2t-18}{}^{\tau}]^{\frac{1-\alpha_{Wife}}{\tau}}\Big\}^{\frac{1}{(\alpha_{Wife}+\chi-\chi\cdot\alpha_{Wife})}} \cdot C_{Wife,t}{}^{\frac{\alpha_{Wife}}{(\alpha_{Wife}+\chi-\chi\cdot\alpha_{Wife})}} / [\beta_{Wife}^{t-18} \cdot (C_{Child,t} + E_t - S_t) + \beta_{Wife}^{t-19} \cdot (C_{Child,t+1} + E_{t+1} - S_{t+1}) + \cdots + \beta_{Wife}^{0} \cdot (C_{Child,2t-18} + E_{2t-18} - S_{2t-18})]^{\frac{1}{(\alpha_{Wife}+\chi-\chi\cdot\alpha_{Wife})}}$$

The probability of having a child in the current period $P(K_t = 1)$ is positively correlated with the qualities of all the current and previous children $\{Q_j\}_{j=t}^{j=2t-18}$, wife's consumption $C_{Wife,t}$, all the current and previous child subsidies $\{S_j\}_{j=t}^{j=2t-18}$; while negatively correlated with all the current and previous child consumption $\{C_j\}_{j=t}^{j=2t-18}$ and child education expenses $\{E_j\}_{j=t}^{j=2t-18}$.

The optimal number of children N_t is independent of wife's discount factor β_{Wife}, however, the probability of having a child in the current period $P(K_t = 1)$ is positively correlated with wife's discount factor β_{Wife}. Lower discount factor in terms of lower value of β_{Wife} and higher risk aversion in terms of higher value of α_{Wife} increases the probability of having a child in the current period $P(K_t = 1)$.

4. Empirical Analysis

4.1 Multinomial Logit Model

Reference category is 'Very Unlikely'=0

Discrete Variable 'Likely to have One or More Children' has 13 scales with higher scale indicating more likely to have one or more children.
Discrete Variable 'General Health Level' has 5 scales with lower scale indicating better health level.

Discrete Variable 'Highest Education Level Achieved' has 9 scales with lower scale indicating higher education achieved.

Discrete Variable 'Do fair share of looking after children' has a 10 scales with lower scale indicating does much more than fair share and higher scale indicating does much less fair share.

Discrete Variable 'Financial Risk Prepared to Take' has a 5 scales with lower scale indicating more willing to take financial risk and higher scale indicating less willing to take financial risk
Discrete Variable 'Looking After Children is More Work than Pleasure' has a 16 scales with lower scale indicating strongly disagree and higher scale indicating strongly agree.

Discrete Variable 'Mother Earns the Money' has a 16 scales with lower scale indicating strongly disagree and higher scale indicating strongly agree.

Discrete Variable 'Working has Positive Effects on Children' has a 16 scales with lower scale indicating strongly disagree and higher scale indicating strongly agree.

'Hours spent on Playing with Children', 'Financial Income for Child Care', 'Marriage Duration' and 'Age' are continuous variables.

4.1.1 Multinomial Model for Females' Likelihood to Have One or More Children

Table 3: Multinomial Model for Females' Likelihood to Have One or More Children

Likely to have one or more children	Hours spent on playing with children	Do fair share of looking after children	Financial income for child care	General health level	Highest education level achieved	Marriage duration	Financial risk prepared to take	Age	Looking after children is more work than pleasure	Mother earns the money	Working has positive effects on children
-4 (Refused , Not stated)	-0.1243	-1.4972	-0.0254	0.5041	0.1095	0.0562	-0.4047	-0.1092	-0.0607	-0.1898	-0.9720
-3(Don't know)	-0.0020	0.4375	-0.0382	0.2950	-0.0959	0.1517 **	0.4883	-0.0006	-0.0835	0.5290	-1.0761
-1(Not asked)	-0.0696 ***	-0.0174	-0.0007 ***	-0.4868 ***	0.1990 ***	0.0002	0.2130 ***	0.0581 ***	-0.1723 ***	0.0288 *	0.1487 ***
1	0.0082 **	0.0798	-0.00002	-0.0311	0.1785 ***	0.0391 ***	-0.0299	-0.0406 ***	. -0.0371	-0.0361	0.0019
2	0.0098 **	-0.0227	-0.0001	-0.1115	0.1373 ***	0.0406 ***	0.0359	-0.0626 ***	0.0653	-0.0161	0.0423
3	0.0145 ***	0.0684	-0.00004	-0.0141	0.1256 ***	0.0584 ***	-0.0773	-0.0760 ***	-0.0360	-0.0089	0.0019 **
4	0.0123 *	-0.1034	-0.0002	-0.1305	0.1953 ***	0.0816 ***	0.2175 **	-0.0768 ***	-0.0839	-0.0717	0.0008 **
5	0.0115 ***	0.0762	0.0001	-0.0758	0.2006 ***	0.0745 ***	0.0565	-0.0835 ***	-0.0873 *	-0.0244	0.0982 ***
6	0.0125 **	0.1220	-0.0004	-0.0499	0.2971 ***	0.0952 ***	0.0577	-0.1013 ***	-0.1053	-0.0616	0.0935
7	0.0077	0.0521	-0.0001	-0.0722	0.3465 ***	0.0878 ***	0.0136	-0.1256 ***	-0.1226	0.0276	0.0582
8	0.0097 **	0.1162	-0.0001 *	-0.1306 *	0.2793 ***	0.1087 ***	0.0457	-0.1216 ***	-0.1460 **	-0.0138	0.0919 **
9	0.0112 **	0.1865	-0.0001	.11586	0.3375 ***	0.1323 ***	0.0195	-0.1312 ***	-0.3517 ***	0.0420	0.0308
10 (Very Likely)	0.0125 ***	0.1124 **	-0.0001 ***	0.1973 ***	0.2647 ***	0.0741 ***	0.0269	-0.1326 ***	. -0.1343 ***	0.0043	0.0741 ***

Note: Reference Category is 0. * Indicates significant at 10%, ** indicates significant at 5%, *** indicates significant at 1%

$$\text{Likely to have }\widehat{\text{One or}}\text{ More Children}_t = \widehat{\vartheta_0} + \widehat{\vartheta_1}\cdot\text{Hours spent on Playing with Children}_t + \widehat{\vartheta_2}\cdot\text{Do fair share of looking after children}_t + \widehat{\vartheta_3}\cdot\text{Financial Income for Child Care}_t + \widehat{\vartheta_4}\cdot\text{General Health Level}_t + \widehat{\vartheta_5}\cdot\text{Highest Education Level Achieved}_t + \widehat{\vartheta_6}\cdot\text{Marriage Duration}_t + \widehat{\vartheta_7}\cdot\text{Financial Risk Prepared to Take}_t + \widehat{\vartheta_8}\cdot\text{Age}_t + \widehat{\vartheta_9}\cdot\text{Looking After Children is More Work than Pleasure}_t + \widehat{\vartheta_{10}}\cdot\text{Mother Earns the Money}_t + \widehat{\vartheta_{11}}\cdot\text{Working has Positive Effects on Children}_t$$

The multinomial model for females' likelihood to have one or more children suggests that compare with the base category 'very unlikely to have one or more children', the more hours that a female would like to spend with children, the more financial income for child care, the healthier the female is, the less education level that a female has, the longer marriage duration expected, the younger the female is, the lower expectation that looking after children is more work than pleasure, the higher expectation that working has positive effects upon children, the more likely a female is going to have one or more children, and these relationships are significant. However, the expectation of fair share of looking after children, financial risk prepared to take and the expectation that mother earns the money do not have significant impacts upon a female's likelihood to have one or more children.

4.1.2 Multinomial Model for Males' Likelihood to Have One or More Children

Table 4: Multinomial Model for Males' Likelihood to Have One or More Children

Likely to have One or More Children	Hours spent on Playing with Children	Do fair share of looking after children	Financial Income for Child Care	General Health Level	Highest Education Level Achieved	Marriage Duration	Financial Risk Prepared to Take	Age	Looking After Children is More Work than Pleasure	Father's Involvement	Working has Positive Effects on Children
-4 (Refused, Not stated)	0.0133	-0.2323	-0.0007	0.9908	14.3939	-2.7375	0.0551	-0.0052	-0.3076	14.8067	-0.6765
-3(Don't know)	-0.0892	-0.2091	-0.0558	0.2592	-0.1916	0.0244	0.2044	-0.1135 ***	0.3923 **	-0.0358	-0.4875
-1 (Not asked)	-0.0551 ***	-0.2292 ***	-0.0001	-0.1510 ***	0.1992 ***	0.0185 ***	0.2643 ***	0.0465 ***	-0.0180	0.0511 ***	-2.1359 ***
1	0.0199 ***	-0.1116 *	0.0001	0.0186	0.1627 ***	0.0548 ***	-0.1389 ***	-0.0161 **	0.0085	0.0687	0.0486
2	0.0133 **	-0.0538	0.0002	-0.0628	0.1245 ***	0.0731 ***	-0.0165	-0.0325 ***	0.0653	0.0098	0.0524
3	0.0201 **	-0.1082	0.0001	-0.0144	0.1932 ***	0.1273 ***	-0.0516	-0.0428 ***	0.0527	0.0067	0.0257
4	0.0196 *	-0.2075 *	0.0439	-0.2403 *	0.1601 ***	0.1278 ***	-0.0806	-0.0417 ***	0.1219	0.0432	0.0432
5	0.0188 ***	-0.0110	0.0002	-0.0664	0.1559 ***	0.1205 ***	-0.0019	-0.0527 ***	-0.0557	0.0472	0.0687 *
6	0.0107	-0.0961	0.0017	-0.0216	0.2347 ***	0.1208 ***	-0.0009	-0.0679 ***	-0.0217	0.0414	0.0131
7	0.0118	-0.0482	0.0517	-0.1186	0.2887 ***	0.1585 ***	-0.0171	-0.0906 ***	-0.0842	0.0683	-0.0665
8	0.0124	-0.0168	0.0575	-0.1563 **	0.2855 ***	0.1366 ***	0.0370	-0.0917 ***	0.0084	0.0406	0.1818 ***
9	0.0032	-0.0138	0.0576	-0.1483	0.3594 ***	0.1339 ***	-0.0418	-0.1203 ***	-0.0368	0.0661	-0.3039
10 (Very Likely)	0.0159 ***	-0.0703	0.0050	-0.1821 ***	-0.2459 ***	0.0955 ***	-0.0688 **	-0.0949 ***	-0.1370 ***	0.0915 ***	-0.6509 *

Note: Reference Category is 0. * Indicates significant at 10%, ** indicates significant at 5%, *** indicates significant at 1%

$$\widehat{\text{Likely to have One or More Children}}_t = \widehat{\delta_0} + \widehat{\delta_1} \cdot \text{Hours spent on Playing with Children}_t + \widehat{\delta_2} \cdot \text{Do fair share of looking after children}_t + \widehat{\delta_3} \cdot \text{Financial Income for Child Care}_t + \widehat{\delta_4} \cdot \text{General Health Level}_t + \widehat{\delta_5} \cdot \text{Highest Education Level Achieved}_t + \widehat{\delta_6} \cdot \text{Marriage Duration}_t + \widehat{\delta_7} \cdot \text{Financial Risk Prepared to Take}_t + \widehat{\delta_8} \cdot \text{Age}_t +$$

$$\widehat{\delta_9} \cdot \text{Looking After Children is More Work than Pleasure}_t + \widehat{\delta_{10}} \cdot \text{Father's Involvement}_t + \widehat{\delta_{11}} \cdot \text{Working has Positive Effects on Children}_t$$

The multinomial model for males' likelihood to have one or more children suggests that compare with the base category 'very unlikely to have one or more children', the more hours that a male would like to spend with children, the more responsibility a man would like to take in looking after children, the more financial income for child care, the healthier the male is, the less education level that a male has, the longer marriage duration expected, the more willing the male is prepared to take financial risk, the younger the male is, the lower expectation that looking after children is more work than pleasure, the more the father would like to be involved in child rearing, the higher expectation that working has positive effects upon children, the more likely a male is going to have one or more children, and these relationships are significant. However, financial income for childcare does not have significant impacts upon a male's likelihood to have one or more children. This also indicates that males are more willing to take risks in child rearing.

4.1.3 Multinomial Model for Females' Number of Children

Table 5: Multinomial Model for Females' Number of Children

Number of children	Hours spent on playing with children	Do fair share of looking after children	Financial income for child care	General health level	Highest education level achieved	Marriage duration	Financial risk prepared to take	Age	Looking After children is more work than pleasure	Mother earns the money	Working has positive effects on children
1	0.1892 ***	0.1157 *	0.0007 ***	-0.1738 ***	0.1300 ***	0.0510 ***	-0.2734 ***	0.0833 ***	0.2791 ***	-0.2084 ***	0.1139 ***
2	0.1821 ***	0.0712	0.0007 ***	-0.0944 **	0.1579 ***	0.0821 ***	-0.3367 ***	0.1071 ***	0.3645 ***	-0.1372 ***	0.1002 ***
3	0.1799 ***	0.0483	0.0008 ***	-0.0792	0.2060 ***	0.0914 ***	-0.3397 ***	0.1114 ***	0.3746 ***	-0.1483 ***	0.1067 ***
4	0.1794 ***	0.0970	0.0008 ***	-0.1389 **	0.1998 ***	0.0895 ***	-0.2854 ***	0.1226 ***	0.3621 ***	-0.1350 ***	0.0142
5	0.1855 **	0.0901	0.0008 ***	-0.2054 ***	0.2656 ***	0.0968 ***	-0.2827 ***	0.1336 ***	0.4075 ***	-0.2206 ***	0.0620
6	0.1899 ***	0.0823	0.0008 ***	-0.2081 **	0.1945 ***	0.0860 ***	-0.2446 ***	0.1555 ***	0.4495 ***	-0.1940 ***	0.0626
7	0.1801 ***	-0.0834	0.0008 ***	-0.2754 *	0.4472 ***	.100386 ***	-0.2823 ***	0.1302 ***	0.5602 ***	-0.1720 ***	-0.0318
8	0.2064 ***	-0.2091	-0.0254	-0.3341 *	0.3123 ***	0.0992 ***	-0.1913 *	0.1575 ***	0.3652 **	-0.2518 ***	0.1423
9	0.1837 ***	-0.5645 *	-0.0419	-0.3931	0.4453 **	0.0689 ***	-0.1065	0.1374 ***	0.3746	-0.2660 **	-0.8835 *
10	0.1739 *	-0.0637	-0.0355	-0.1694	2.6949	0.0918 ***	2.1639 *	0.1782 ***	0.1333	-0.2030	-0.5835
11	-0.0092	-1.4822 **	0.0008	-1.0274	9.6604	1.0220 **	-0.0781	-0.6233	0.1752	-0.1490	0.4099
12	-0.0608	-1.1940 **	-0.0080	-2.8094	0.1346	0.0249	19.7758 ***	0.1260 *	0.7037 **	-0.0211	-0.5741 *
13	-0.1887	-0.6705	0.0008 ***	1.5583 **	3.1685	0.0595 *	0.6229	0.0926 **	0.4322	-0.2648	-1.8633

Note: Reference Category is 0. * Indicates significant at 10%, ** indicates significant at 5%, *** indicates significant at 1%

$$\widehat{\text{Number of Children}}_t = \widehat{\alpha_0} + \widehat{\alpha_1} \cdot \text{Hours spent on Playing with Children}_t + \widehat{\alpha_2} \cdot \text{Do fair share of looking after children}_t + \widehat{\alpha_3} \cdot \text{Financial Income for Child Care}_t + \widehat{\alpha_4} \cdot \text{General Health Level}_t + \widehat{\alpha_5} \cdot \text{Highest Education Level Achieved}_t + \widehat{\alpha_6} \cdot \text{Marriage Duration}_t + \widehat{\alpha_7} \cdot \text{Financial Risk Prepared to Take}_t + \widehat{\alpha_8} \cdot \text{Age}_t + \widehat{\alpha_9} \cdot \text{Looking After Children is More Work than Pleasure}_t + \widehat{\alpha_{10}} \cdot \text{Mother Earns the Money}_t + \widehat{\alpha_{11}} \cdot \text{Working has Positive Effects on Children}_t$$

The multinomial model for females' number of children suggests that compare with the base category '0 children', the more hours that a female would like to spend with children, the more financial income for child care, the more responsibility would like to take in looking after children, the healthier the female is, the less education level that a female has, the longer marriage duration expected, the older the female is, the higher expectation that looking after children is more work than pleasure, the lower expectation that mother earns the money, the higher expectation that working has positive effects upon children, the more number of children a female is likely to already have, and these relationships are significant.

4.1.4 Multinomial Model for Males' Number of Children

Table 6: Multinomial Model for Males' Number of Children

Number of Children	Hours spent on playing with children	Do fair share of looking after children	Financial income for child care	General Health Level	Highest education level achieved	Marriage duration	Financial risk prepared to take	Age	Looking after children is more work than pleasure	Father's involvement	Working has positive effects on children
1	0.1660 ***	0.3286 ***	0.0034	0.1969 ***	0.0327	0.0607 ***	-0.3224 ***	0.0735 ***	0.0589 ***	-0.1530 ***	0.1432 ***
2	0.1609 ***	0.2613 ***	0.0040	0.1386 ***	0.0076	0.0921 ***	-0.3552 ***	0.0935 ***	0.0865 ***	-0.1270 ***	0.2069 **
3	0.1562 ***	0.3156 ***	0.0037	0.1053 **	0.0482 ***	0.0992 ***	-0.3498 ***	0.1069 ***	0.1237 **	-0.1544 ***	0.1618 ***
4	0.1487 ***	0.3356 ***	0.0039 *	0.1279 **	0.0336	0.1000 ***	-0.3319 ***	0.1249 ***	0.1265 **	-0.1739 ***	0.1884 ***
5	0.1649 ***	0.2717 **	-0.0317 ***	0.3094 ***	0.1385 ***	0.1035 ***	-0.3633 ***	0.1277 ***	0.2725 ***	-0.1690 ***	0.0583
6	0.1841 ***	0.2813 *	0.0034	0.3596 **	0.0424	0.1003 ***	-0.3796 ***	0.1516 ***	0.0799	-0.2062 ***	0.2982 ***
7	0.1642 ***	0.4632 **	-0.0304	0.6810 ***	0.1842 ***	0.1042 ***	-0.2370 **	0.1092 ***	-0.0006	-0.1286 *	-0.1754
8	0.2031 ***	0.7110 ***	-0.0251	0.7284 ***	0.1686	0.1060 ***	-0.5010 *	0.1593 ***	-0.2788	-0.1280	0.2409
9	0.1568 ***	0.2659	-0.0660	0.2204 ***	0.7878	0.0689 ***	-0.0041	0.1584	0.3746	-0.2341	0.4216 *
10	0.1913 ***	0.0251	-0.1279	-0.2686	0.0571	0.0883 ***	0.6666	0.2208 ***	0.7077	0.5615	-3.5337 ***
11	0.2733	39.1048	0.0702	68.2026	-23.0761	32.9572	1.0645	-26.582	-14.2090	-14.656	17.7613
14	0.1931	0.4665	0.0070	1.2398	13.8973 **	0.1040	-0.7757	0.3362 *	0.1125 *	14.668	-2.4525

Note: Reference Category is 0. * Indicates significant at 10%, ** indicates significant at 5%, *** indicates significant at 1%

$$\widehat{\text{Number of Children}}_t = \widehat{\gamma_0} + \widehat{\gamma_1} \cdot \text{Hours spent on Playing with Children}_t + \widehat{\gamma_2} \cdot \text{Do fair share of looking after children}_t + \widehat{\gamma_3} \cdot \text{Financial Income for Child Care}_t + \widehat{\gamma_4} \cdot \text{General Health Level}_t + \widehat{\gamma_5} \cdot \text{Highest Education Level Achieved}_t + \widehat{\gamma_6} \cdot \text{Marriage Duration}_t + \widehat{\gamma_7} \cdot \text{Financial Risk Prepared to Take}_t + \widehat{\gamma_8} \cdot \text{Age}_t + \widehat{\gamma_9} \cdot \text{Looking After Children is More Work than Pleasure}_t + \widehat{\gamma_{10}} \cdot \text{Father's Involvement}_t + \widehat{\gamma_{11}} \cdot \text{Working has Positive Effects on Children}_t$$

The multinomial model for males' number of children suggests that compare with the base category '0 children', the more hours that a male would like to spend with children, the less responsibility a man would like to take in looking after children, the more financial income for child care, the healthier the male is, the less education level that a male has, the longer marriage duration expected, the less willing the male is prepared to take financial risk, the older the male is, the higher expectation that looking after children is more work than pleasure, the less the father would like to be involved in child rearing, the higher expectation that working has positive effects upon children, the more number of children a male is likely to already have, and these relationships are significant. However, financial income for childcare does not have significant impacts upon a male's number of children.

5. Conclusions

The theoretical household decision making model and the empirical analysis using Australian data suggest that husband and wife differs in fertility decision making due to the difference in risk aversion, and this influences husband's and wife's decision making in the optimal number of children to produce, the optimal household consumption, the optimal division of labour between working and child rearing. Wives are assumed to be more risk averse than husbands; The household decision making process is modelled by specifying husband's and wife's utilities as functions of household consumption, the number of dependent children in the family and average quality of children; the fertility decision making process is simulated by maximizing household's utility subject to wealth constraints; the fertility decision making environment is characterized by couple's wages, child care subsidies, inflation, education expenditure and health conditions. The following conclusions are obtained.

(1) Wives are less likely to make risky fertility decisions such as choosing to produce a child in the circumstances of financial disadvantages or health challenges.

(2). Wages have double effects upon fertility decision making, higher wages enable couples to raise more children. However, higher wages also increase the opportunity costs of the labour spending in raising children.

(3). The division of labour between working and child rearing has double effects upon the average quality of children in a family, more labour spent in working help to increase the average quality of dependent children at the cost reducing the labour spent in child rearing, which tends to decrease the average quality of dependent children.

(4). Wage difference between husband and wife plays a more important role in high income families' fertility decision making than low income families' fertility decision making. Higher family income leads to more education expenditure on children.

High income families where husband earns much more than wife tend to produce more children, provide more parental care and more education expenditure for children, since wife's opportunity cost for child rearing is relatively low, wife can spend more time on child rearing which helps to increase the quality of children.

High income families where wife earns much more than husband tend to produce less children, but provide more parental care and more education expenditure for children, since husband's opportunity cost for child rearing is relatively low, husband can spend more time on child rearing which helps to increase the quality of children. However, this type of family usually produces fewer children since the opportunity cost for wife's child bearing is high.

High income families where wage difference between husband and wife is small tend to produce less children, provide less parental care but more education expenditure for children, since both of the couple's opportunity cost for child rearing is high, they tend to spend less time on child rearing but are able to spend more money on education for children.

Low income families tend to produce more children, provide more parental care and provide less education expenditure for children, since both of the couple's opportunity costs for child rearing are low, they tend to produce more children and obtain more child care subsidies from the government.

(5). The changes of wage difference between husband and wife over time, the dynamics of average household income and the dynamics of child care subsidies can explain some features of Australia's fertility rate from 1990 to 2013, lower average family incomes and higher wage differences seem to be associated with higher fertility rates, however, higher average family incomes and lower wage differences do not seem to generate impacts upon fertility rates.

These findings support the hypothesis that husbands and wives' optimal fertility decisions differ in direction and magnitude due to heterogeneous utility functions, household income, inflation and child welfare policies. These findings also shed light on policy makers that although couples' decision making processes are subject to biological constraints, policy makers can influence their optimal fertility choices by changing their financial constraints in terms of inflation, wages, taxes and welfare.

The limitations of this paper are the following. Firstly, the theoretical model assumes couples have time invariant preference and risk aversion, which are independent of their infertility risk, the number and quality of their existing children; however, these contract the reality that couples' preference and risk aversion may change in accordance with their infertility risk, the number and quality of their existing children. Secondly, the empirical analysis models couples' decision processes of the likelihood to have children and the number of children to have separately, however, the two decision processes can be integrated into the same model to allow for their interdependence. Hence, further research and extension should incorporate these factors into modelling households' fertility decision making process.

Endnotes

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[i] Husband's relative risk aversion= $R(c_{\text{Husband}})=\frac{-c_{\text{Husband}}\cdot u_{\text{Husband}}''(c)}{u_{\text{Husband}}'(c)}$

$$= -c_{\text{Husband}} \cdot \frac{-\alpha_{\text{Husband}}(1-\alpha_{\text{Husband}})c_{\text{Husband}}^{-\alpha_{\text{Husband}}-1}}{1-\alpha_{\text{Husband}}} \cdot \frac{1-\alpha_{\text{Husband}}}{(1-\alpha_{\text{Husband}})c_{\text{Husband}}^{-\alpha_{\text{Husband}}}} = \alpha_{\text{Husband}}$$

[ii] Wife's relative risk aversion= $R(c_{\text{Wife}})=\frac{-c_{\text{Wife}}\cdot u_{\text{Wife}}''(c)}{u_{\text{Wife}}'(c)}$

$$= -c_{\text{Wife}} \cdot \frac{-\alpha_{\text{Wife}}(1-\alpha_{\text{Wife}})c_{\text{Wife}}^{-\alpha_{\text{Wife}}-1}}{1-\alpha_{\text{Wife}}} \cdot \frac{1-\alpha_{\text{Wife}}}{(1-\alpha_{\text{Wife}})c_{\text{Wife}}^{-\alpha_{\text{Wife}}}} = \alpha_{\text{Wife}}$$

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