

Development of Inequality: A Theoretical Treatise

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Income inequality is one of the most discussed topics of development economics. Starting from Kuznets (1955) many studies have talked about the evolution pattern of inequality through human society. Political dimensions along with economic determinants are used in literature to explain variations of inequality. Among political issues some argue that free market economy and the declining strength of socialist arguments in policy making have meant income inequality continues to rise. In economic front issues like human capital (and education) and fertility rates are argued to be the key determinants of income inequality. However, other studies (Rahman and Senan, 2014) have found the incidence of education and fertility in developing countries to be an outcome of income inequality, and not the other way around. By starting off with homogenous families in terms of endowments, and varying in their preferences, this current paper will attempt to develop a theoretical model showing how income inequality may evolve in an otherwise equal society; and its implications; and the effect redistributive practices may have on the evolution of income inequality.

Keywords: Income inequality, Education, Bequest

1. Introduction

Income inequality is present, to some degree, across all human societies. However, there has been a continuous rising trend in income inequality in the last five or six decades (Piketty, 2013, 2014; Sala-i-Martin, 2005). Furthermore, in recent times, a greater shift towards free, market economy and the declining strength of socialist arguments in policy making have meant income inequality continues to rise (Milanovic & Squire, 2005; Easterly, 2007). Apart from income inequality is measured by dispersion of wealth distribution which has its own effectiveness (Smeeding & Jeffrey, 2011).

Some of the key factors that lead to emergence of income inequality are variation within the population in incidence of education, fertility rates, and wealth. In recent years, a few papers have tried to develop a unified and endogenous model with all these factors (de la Croix & D, 2003; Doepke, 2004; Galor & DN, 2000; Hansen & Prescott, 2002; Kremer & Chen, 2002; Sato, et al., 2008, Becker, Murphy, and Tamura, 1994; Torvik, 1993). Their arguments are as follows: parents decide between having a high number of children, or fewer children with higher human capital by investing more in their education. Since education is directly related to earning potentials, inequality develops between groups with a large number of agents with low human capital and a small number of agents with high

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human capital. If there exists high enough level of inter-generational persistence in preference, agents with high human capital will have high preference to educate their children more, vice versa. Overtime, the (opportunity) cost of education declines, leading to higher incidence of education and reversal of income inequality trend. Other studies have looked into the impact of intergenerational inheritance on inequality (Miyazawa, 2006; Piketty, 2013).

In this paper, we will attempt to develop a theoretical framework with endogenous fertility, savings, education, and inheritance decisions made by agents. By running simulations on a closed society with homogenous economic agents in terms of wealth and human capital endowments to start with, and varying in their preferences (which is persistent across generations), we will chart the evolution of income inequality over multiple generations. We will contrast this finding by comparing it with a scenario where the preference structure of agents are not constant but may change (through exogenous factors we will not delve into).

Certain developed nations have managed to efficiently tackle high income inequality through efficient public welfare policies (high quality public education and health care, high and/or progressive taxation, et cetera). Such schemes have, however, failed to bring about much positive outcome in bulk of the developing nations. On the flipside, trickle-down theory, equity-efficiency theory, et cetera, have argued for greater emphasis on growth and economic development but that has only further exacerbated the income inequality situation in the developing nations. In this paper, we will also look into the affects that redistributive practices can have on the overall income inequality scenario of the society. Notably, we will focus on the impact of progressive redistribution on inequality.

The distribution of this paper is as such: in section 2, we discuss some of the recent and relevant studies dealing with the issue of income inequality; in section 3, we will present the theoretical framework of our model; in section 4, we will run simulations and present the implications of the model; in section 5, we will briefly discuss the policy implications of the findings from our model; and in section 6, we will make some concluding remarks. This will be followed by appendices and a bibliography.

2. Review of Relevant Literature

There is a consensus in relevant literature that a perfectly egalitarian society may not be feasible, or even desirable. The seminal paper by Kuznets (Kuznets, 1955) first proposed the idea that a little inequality maybe necessary for growth to occur (or rather, for growth to take-off) in the initial phases of development. The equity-efficiency argument has sided with Kuznets, and argued that no growth is possible without inequality (Okun, 1975) and that if a state tries too hard to bring income parity through pro-transfer and redistributive policies, they are going to discourage capital accumulation and growth. Another school of thought has argued that high income inequality is not only detrimental to growth, but also immoral and unfair (Persson & Tabellini, 1994; Alesina & Rodrik, 1994). This school argues for a negative relationship between inequality and growth (Stevans, 2012). Stevans (2012) further found that equal distribution can lead to growth too and found no evidence that if a country tried too hard to redistribute income to the lower quintiles, capital accumulation will be harmed.

Derivation of an optimum level of income inequality (zero or otherwise) has not yet been theorized. In the absence of that toolkit, we are best served by identifying the determinants

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of income inequality and then charting its trend overtime and then relying on our judgment to tweak our redistributive policies to move towards the desired level of income inequality.

An interesting result came out following Kuznet's inverted-U argument following Banarjee & Duflo (2003). Using a simple 'hold-up' model they showed that if one group (usually the poor since their share of resource in the economy is less and thus, has less at stake) can hold-up another (richer) group and demand a transfer before letting a policy (which would cause growth) being passed, such a transfer, which would reduce inequality, would be significantly less than the growth that could have been realized had no hold-up taken place.

Of course, the nature of transfer will largely depend upon the political system, or the ideology, of the state. Several researches have managed to show a relationship between type of political regime and income inequality (Benabou, 2000; Bourguignon & Verdier, 2000; Verardi, 2005) and between the type of institutions and income inequality (Chong & Calderon, 2000; Chong, 2004; Chong & Gradstein, 2007; Sunde, et al., 2008; Easterly, 2007), and the effect of redistribution in democracies.

Different initial condition in people's interpretation of 'fair', 'just' and 'acceptable level of inequality' but otherwise identical in terms of resource endowment and wealth distribution can put the countries on different growth paths for an extended period of time (Alesina, et al., 2009). With high enough persistence of ideological beliefs between generation, transfer decisions (level of taxation) are made based on the historic mind frame of its people, irrespective of current economic conditions. These different 'ideologies' of different countries are shaped by various factors. A history of misfortune and repression in the formative years, or even growing up during a recession, would turn people more risk-averse. These people tend to believe that success in life depends more on luck than effort, support more government redistribution, yet, at the same time, are less confident in public institutions and these beliefs are long-lasting. Following from that line of argument, poor people would prefer higher transfer (since their future expectations won't be very optimistic). Following the same line of argument, rich people would prefer less transfer from them as they too would expect a grim future for themselves. A history of good fortune, obviously, would have the reverse effect.

Few have argued that redistributive spending by the government that is financed by taxation puts a negative spin on capital accumulation and investment (Alesina & Rodrik, 1994). This leads to a stagnating growth. Contrastingly, others argue that when government aids such as welfare, pension, subsidies, grants, et cetera, are in place and active, the economy will grow (Persson & Tabellini, 1994), even though inequality will worsen. They also found a positive relationship between income equality and growth only in democratic countries, implying that impact of (in)equality on growth most probably only work through political channels. Similar findings were reported by Hsu (2008). By dividing worlds states into communist regimes, Islamic republics, social democracies, European colonies, military dictatorship, conservative democracy and dictatorship, Hsu (2008) tested the impact of different type political regimes on income inequality between 1963 and 2002.

Other studies have argued that countries that experience high income inequality also tends to spend less on redistribution (Mello & Tiongson, 2006). Thus, the governments of countries that suffer the most from low GDP per capita and high income inequality are also less likely to redistribute income through their public policies. Inequality will exist unchallenged in countries that are suffering the most due to it.

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Any discussion on redistribution will also have to include the issue of tax avoidance. Even though we will skirt around this issue in our model, and assume honest reporting of income and wealth levels by individuals, the situation is far from ideal in reality. High level of progressive taxation may lower observed income inequality, but actual income inequality will increase (Duncan & Peter, 2012). Furthermore, since the gap between actual and observed inequality is increasing in tax, countries that is trying to implement high levels of redistribution will suffer the most from tax avoidance and their redistributive policies will also be the least efficient. Furthermore, income from capital is easier to hide than income through wages, which makes it easier for wealthy people to hide their incomes than poorer people, further exacerbating the income inequality situation.

3. Theoretical Framework

This section presents the basic structure of the model and its underlying assumptions. The economy has a number of homogenous economic agents in terms of wealth and human capital endowment. Each agent works for a fixed level of wage, w_t . Total income for period t is given by:

$$\text{Income, } I_t = w_t l_t h_t$$

l is the labor unit devoted to income earning activity and h is the human capital. Maximum income for the period is wh , when the entire labor unit is devoted to income generating activity.

$$l_t = 1 - \eta_t(e_t + \tau); \tau \in (0,1)$$

η_t is the number of children an agent decides to have; τ is the portion of time needed to be spent behind child-rearing. Therefore, $(\frac{1}{\tau} - 1)$ is the maximum number of children an agent can have in situations when inheritance and redistribution by the state is zero.

e_t is the time the agent spends providing education to each of his children. Another alternate explanation can be that e_t is the portion of maximum potential income that is spent behind education of children.

$$h_t = 1 + \alpha h_{t-1} + \beta e_{t-1}^\varepsilon; \alpha, \varepsilon \in (0,1), \beta \geq 1$$

Human capital is assumed to depend upon the education received by the agents in the previous period and the human capital of parents from the previous generation. All agents have a minimum human capital of 1. This ensures that even when $e_t = 0$, human capital continues to grow. This can be called human capital accumulation through practical experience, rather than from formal education. However, this does not go on indefinitely, and human capital from practical experience faces an upper bound at $\frac{1}{1-\alpha}$.

Agents maximize their utility in each period and the utility function is given by the following expression:

$$U_t = a \ln(C_t) + b \ln(\eta_t \cdot e_t^\varepsilon) + c \ln(B_{t+1})$$

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C_t is the consumption of period t , and is equal to $(1 - s)(I_t + B_t)$, where s is the savings rate (MPS) and B_t is the bequest earnings of period t , coming from period $(t - 1)$. a is the preference coefficient assigned to consumption. Consumption is constrained by the inequality, $C_t \geq \bar{C}$, where \bar{C} is a positive constant, representing the autonomous part of consumption.

$\eta_t \cdot e_t^\varepsilon$ is the quality-quantity nexus of children, and parents decide whether they want high number of children with low human capital or low number of children with high human capital. b is the preference coefficient assigned to the quality-quantity continuum of children.

$B_{t+1} = \frac{s(I_t + B_t)}{\eta_t}$ is the level of bequest being left back for each agent for the next period and r is the interest rate between period t and period $(t + 1)$. c is the coefficient assigned as bequest preference.

3.1 Baseline Model

Agents' optimization decision is given by:

$$\begin{aligned} \operatorname{argmax}\{\eta_t, e_t, s\} \\ &= a \ln[(1 - s)\{w_t h_t - w_t h_t \eta_t (e_t + \tau) + B_t\}] + (b - c) \ln(\eta_t) + \varepsilon \cdot b \ln(e_t) \\ &\quad + c \ln[s(1 + r)\{w_t h_t - w_t h_t \eta_t (e_t + \tau) + B_t\}] \\ \text{Subject to: } &(1 - s)[w_t h_t \{1 - \eta_t (e_t + \tau)\} + B_t] \geq \bar{C} \end{aligned}$$

Optimization gives us the following results (*workings are given in appendix 1*):

$$\begin{aligned} s^* &\leq \frac{c(w_t h_t + B_t - \bar{C})}{c(w_t h_t + B_t - \bar{C}) + \bar{C}b} \\ \eta_t^* &\leq \frac{(w_t h_t + B_t - \bar{C})\{b(1 - \varepsilon) - c\}}{b\tau w_t h_t} \\ e_t^* &= \frac{\varepsilon \tau b}{\{b(1 - \varepsilon) - c\}} \end{aligned}$$

Some observations from the optimized expressions:

- Savings rate is increasing in income and bequest at a decreasing rate, with an upper limit at 1. High preference for quality or quantity of children lowers savings rate as more is spent behind children in present period, rather than leaving back a high bequest in the next period.
- The multiplicative relationship between e_t and η_t in the utility function makes the optimized expressions of the two moving in opposite directions for changes in some variables. For example, a rise in τ leads to a fall in number of children that each agents decide to have. However, this is compensated by increasing the time spent educating each child. Therefore, families optimize along the quality-quantity continuum for their children.
- An interesting outcome of the optimization is that education decisions depend solely on the preference of agents and other fixed (exogenous) variables. Individuals can

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augment their children's future income by either providing them education, or by savings up and leaving back high bequest amount.

- In section 4, we will chart income inequality (Gini coefficient) arising over 20 generations under this framework. Before that, we present out next framework that includes taxation and redistribution into this model.

3.2 Government Taxation and Redistribution

Now we would like to include progressive taxation into the model. We take a tax-free income-ceiling, Y^T , above which, everyone pays $t \in (0,1)$ rate of tax. Including a tax schedule, with staggered levels of increasing tax-rates for blocks of income will only serve to complicate the model without changing the result qualitatively. So we stick with t only which is sufficient to ensure progressivity due to the inclusion of the tax-free ceiling. Redistribution of tax revenue is done equally to all individuals in the economy. This further strengthens the progressive nature of the fiscal policy, when analyzed together with taxation (at the lowest level, people earning less than the tax-free ceiling have their disposable incomes augmented whereas at the other end, those with the highest income experiences the highest net-decline in their disposable income for the period due to redistribution).

Since redistribution is done equally, we assume that with high enough population in an economy, total tax revenue is sufficiently high and does not enter an agents' optimization decision, i.e., the amount is taken to be exogenous. This is not a strong assumption in the practical sense, but important to keep in mind in this theoretical framework.

With the introduction of redistribution, agents optimize according to their disposable income. The disposable incomes are given by:

$$I^D = \begin{cases} (I_t - Y^T)(1 - t) + Y^T + B_t + \frac{t \sum Y^T}{\sum \eta_{t-1}}, & \text{if } w_t l_t h_t > Y^T \\ I_t + B_t + \frac{t \sum Y^T}{\sum \eta_{t-1}}, & \text{if } w_t l_t h_t \leq Y^T \end{cases} ; \sum \eta_{t-1} = \text{population}_t$$

This gives us the following utility functions: $U = a \ln[(1 - s)I^D] + (b - c) \ln(\eta_t) + \varepsilon \cdot b \ln(e_t) + c \ln[sI^D]$

Optimization will be done subject to: $(1 - s)[w_t h_t \{1 - \eta_t(e_t + \tau)\} + B_t] \geq \bar{C}$

Optimization gives us the following results (the derivation of these follows directly from the previous derivations and need not be discussed in details):

$$s^* \leq \begin{cases} \frac{c \left\{ w h (1 - t) + B + t \cdot Y^T + \frac{t \sum Y^T}{\sum \eta_{t-1}} - \bar{C} \right\}}{c \left\{ w h (1 - t) + B + t \cdot Y^T + \frac{t \sum Y^T}{\sum \eta_{t-1}} - \bar{C} \right\} + b \cdot \bar{C}}, & \text{if } w_t l_t h_t > Y^T \\ \frac{c \left\{ w h + B + \frac{t \sum Y^T}{\sum \eta_{t-1}} - \bar{C} \right\}}{c \left\{ w h + B + \frac{t \sum Y^T}{\sum \eta_{t-1}} - \bar{C} \right\} + b \cdot \bar{C}}, & \text{if } w_t l_t h_t \leq Y^T \end{cases}$$

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$$\eta^* \leq \begin{cases} \frac{\{b(1-\varepsilon) - c\} \left\{ wh(1-t) + B + t.Y^T + \frac{t \sum Y^T}{\sum \eta_{t-1}} - \bar{C} \right\}}{wh.b.\tau(1-t)}, & \text{if } w_t l_t h_t > Y^T \\ \frac{\{b(1-\varepsilon) - c\} \left\{ wh + B + \frac{t \sum Y^T}{\sum \eta_{t-1}} - \bar{C} \right\}}{wh.b.\tau}, & \text{if } w_t l_t h_t \leq Y^T \end{cases}$$

$$e^* = \frac{\varepsilon \tau b}{\{b(1-\varepsilon) - c\}}$$

In section 4, we chart income inequality arising over 20 generations under this framework, when taxation and redistributive practices are under use.

4. Charting Income Inequality

Simulation

Using the values of exogenous variables given in table 1, the evolution of income inequality, across 20 generations, under the baseline model and the model with taxation and redistribution are given in figure 1. Preference coefficient assigned to bequest in the utility function, c is the only factor that differentiates agents at $t = 0$ and this value is taken to be persistent across generations. For the simulation, we assume that c is distributed within the society in 5 strata such that $c \sim N$.

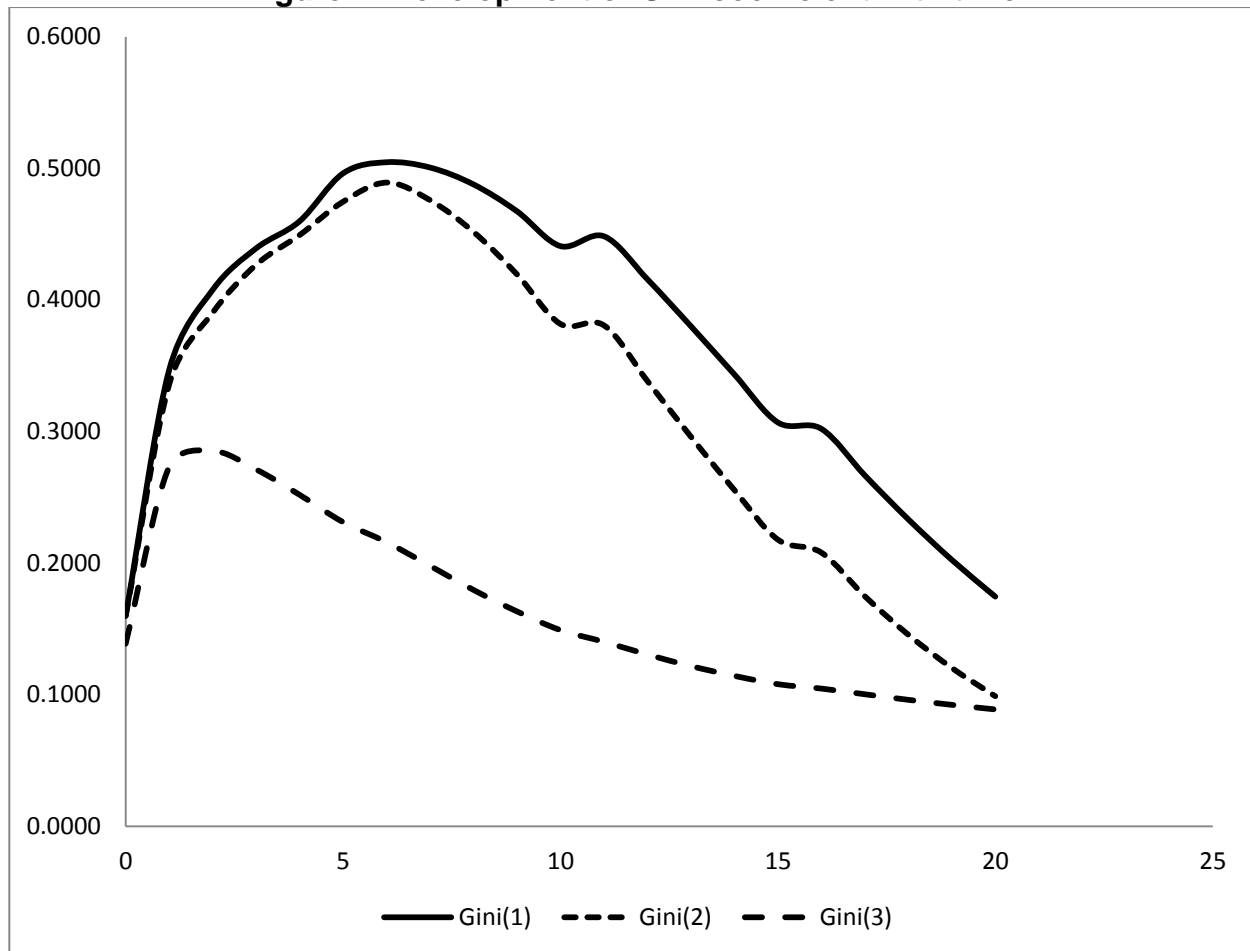
Table 1: Values of Exogenous Variables

a	b	τ	ε	ω	C	α	β	c	t	Y^T
1	1	0.25	0.25	100	25	0.5	5	0-0.6	0.5	100

We assume that in the initial phase, there are P_1 individuals with $c = 0.30$, P_2 individuals with $c = 0.15$, P_2 individuals with $c = 0.45$, P_3 individuals with $c = 0.00$, and P_3 individuals with $c = 0.60$, such that $P_1 > P_2 > P_3$ and $P_1 - P_2 > P_2 - P_3$, and total population, $P = P_1 + 2P_2 + 2P_3$.

Gini(1) charts the inequality of the baseline model (gross gini) and Gini(2) charts the inequality when we include taxation above a tax-free level of income and equal redistribution (net gini). Inequality here is being calculated on both income and inheritance, which presents a clearer picture of the inequality in the society that the orthodox practice of using income. Gini(3), which measures inequality when savings preference is not consistent across generations also lies underneath Gini(1); if poorer people can raise their savings rate, inequality falls down faster than Gini(1), and the trend reversal occurs at an earlier date. Depending on the level of change in savings preference, the curve may rise above or beneath Gini(2).

Figure 1: Development of Gini coefficient with time



The first observation is that the model affirms the existence of Kuznets' curve. Inequality rises rapidly during the initial periods as a small fraction of individuals experience rapid income rise, compared to others. However, overtime, the economy experiences a gradual decline in income inequality as the general level of income of the poorer cohorts rise. With effective redistributive policies in place, this trend-reversal can be sped up in a way that $gross_gini > net_gini$. Therefore, it is possible to reduce income inequality with effective tax and redistribution policies. Awareness raising campaigns that increases the discount-factor of bequest can be used to achieve the same.

This simple minded exercise develops few results of policy importance. Starting from an equal distribution based on child rearing and fertility preference significant inequality can develop which eventually can wane as more and more people express their preference to more sensible option (fewer children with higher level of education). However this process can take a long time while left alone to natural instinct. To expedite this process some bit of transfer may be helpful. This may administered at government level (tax-government spending) or at private level (zakat etc.). Motivational campaign from government to reduce fertility or enhance skill attainment is also more likely to speed up the process (effect not calculated here). Such effect can be secured from other channels as well like remittance. Income generated from foreign countries can effectively ease up income inequality situation. Many developing countries have high level of remittance income which contributes positively in their economy. Proper calibration of parameters can enhance this process at optimal level.

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This exercise however is focused on income inequality only and does not say much about wealth inequality. In reality wealth inequality has been found as more severe than income inequality (Piketty, 2014). Poor people while can earn something but very unlikely to save. In that sense this treatise is to some extent limited.

This study has found that education can improve inequality situation permanently. We have not distinguished between different levels and types of education. This means that if education enhances income potential by helping gathering marketable qualities then that can contribute to distribute income more evenly. This can be done by vocational training as well. Mutual strength of effect of different level of education is actually an empirical issue and can vary from country to country.

Calibration is also needed with pre and post-tax transfer mechanism. While minimum wage, better education can help generate more income for the wage earners, corporate tax, Zakat can help creating opportunities for the group who are waiting in the queue to enjoy financial freedom.

5. Conclusion

This simple exercise brings some important results in more explicit form. Using deductive approach we have found support for Kuznets type inverted U inequality development pattern. We also found that education can improve income inequality situation permanently. Among the internal drivers of the inequality importance of political institutions is never ignored. Education can make a dent in that approach by empowering people and enhancing their income potential. However labor market imperfections need to be addressed in this process. Enhancing workers union and gradual revision of minimum wage can be quite helpful to ease up inequality situation. Tax transfer and other types of distributive approaches can come handy as well. They may work as a measure to reduce wealth gap. However careful attention needs to be given so that the process can generate expected results. Importance of political institutions in generating growth is always highlighted in literature; however, effect of its malfunctioning is also not uncommon. Government help in form of policy and transfer can expedite this process. Private help can also generate such results. This study can be extended in an interesting way by deductively including influences of different elite groups (like political elites or land aristocrats etc.) in decision making and thus observing the development of inequality over time.

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Appendix

$$U_t = a \ln(C_t) + b \ln(\eta_t \cdot e_t^\varepsilon) + c \ln(B_{t+1})$$

Given,

$$C_t = (1 - s)(I_t + B_t)$$

$$I_t = w_t l_t h_t = w_t h_t \{1 - \eta_t(e_t + \tau)\}$$

$$B_{t+1} = \frac{s(I_t + B_t)}{\eta_t}$$

Constraint: $C_t \geq \bar{C}$

$\therefore \operatorname{argmax}\{\eta_t, e_t, s\}$

$$= a \ln[(1 - s)\{w_t h_t - w_t h_t \eta_t(e_t + \tau) + B_t\}] + (b - c) \ln(\eta_t) + \varepsilon \cdot b \ln(e_t) + c \ln[s(1 + r)\{w_t h_t - w_t h_t \eta_t(e_t + \tau) + B_t\}]$$

Subject to: $(1 - s)[w_t h_t \{1 - \eta_t(e_t + \tau)\} + B_t] \geq \bar{C}$

OPTIMIZATION (using Lagrangian)

$$\frac{dL}{d\eta} = -\frac{a(1 - s)wh(\tau + e)}{(1 - s)\{w_t h_t - w_t h_t \eta_t(e_t + \tau) + B_t\} + \lambda_\eta(1 - s)wh(\tau + e) = 0} + \frac{(b - c)}{\eta} - \frac{csw h(\tau + e)}{a\{w_t h_t - w_t h_t \eta_t(e_t + \tau) + B_t\}}$$

$$\therefore \lambda_\eta = \frac{a}{(1 - s)\{w_t h_t - w_t h_t \eta_t(e_t + \tau) + B_t\}} - \frac{(b - c)}{\eta(1 - s)wh(\tau + e)}$$

$$+ \frac{(1 - s)\{w_t h_t - w_t h_t \eta_t(e_t + \tau) + B_t\}}{a(1 - s)wh\eta}$$

$$\frac{dL}{de} = -\frac{a(1 - s)wh\eta}{(1 - s)\{w_t h_t - w_t h_t \eta_t(e_t + \tau) + B_t\} + \lambda_e(1 - s)wh\eta} + \frac{\varepsilon b}{e} - \frac{csw h\eta}{s\{w_t h_t - w_t h_t \eta_t(e_t + \tau) + B_t\}}$$

$$\therefore \lambda_e = \frac{a}{(1 - s)\{w_t h_t - w_t h_t \eta_t(e_t + \tau) + B_t\}} - \frac{\varepsilon b}{e(1 - s)wh\eta}$$

$$+ \frac{(1 - s)\{w_t h_t - w_t h_t \eta_t(e_t + \tau) + B_t\}}{a\{w_t h_t - w_t h_t \eta_t(e_t + \tau) + B_t\}} + \frac{c\{w_t h_t - w_t h_t \eta_t(e_t + \tau) + B_t\}}{s\{w_t h_t - w_t h_t \eta_t(e_t + \tau) + B_t\}}$$

$$\frac{dL}{ds} = -\frac{a\{w_t h_t - w_t h_t \eta_t(e_t + \tau) + B_t\}}{(1 - s)\{w_t h_t - w_t h_t \eta_t(e_t + \tau) + B_t\} + \lambda_s\{w_t h_t - w_t h_t \eta_t(e_t + \tau) + B_t\} = 0} + \frac{c\{w_t h_t - w_t h_t \eta_t(e_t + \tau) + B_t\}}{s\{w_t h_t - w_t h_t \eta_t(e_t + \tau) + B_t\}}$$

$$\therefore \lambda_s = \frac{a}{(1 - s)\{w_t h_t - w_t h_t \eta_t(e_t + \tau) + B_t\}} - \frac{c}{s\{w_t h_t - w_t h_t \eta_t(e_t + \tau) + B_t\}}$$

Since $\lambda_\eta = \lambda_e = \lambda_s$,

$$\begin{aligned} & \left[\frac{(b - c)}{\eta(1 - s)wh(\tau + e)} - \frac{c}{(1 - s)\{w_t h_t - w_t h_t \eta_t(e_t + \tau) + B_t\}} \right] \\ &= \left[\frac{\varepsilon b}{e(1 - s)wh\eta} - \frac{c}{(1 - s)\{w_t h_t - w_t h_t \eta_t(e_t + \tau) + B_t\}} \right] \\ &= \left[\frac{c}{s\{w_t h_t - w_t h_t \eta_t(e_t + \tau) + B_t\}} \right] \dots (1) \end{aligned}$$

For simplification, we can write (1) as:

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$$\left[\frac{(b-c)}{\eta(1-s)wh(\tau+e)} - \frac{c}{(1-s)(I+B)} \right] = \left[\frac{\varepsilon b}{e(1-s)wh\eta} - \frac{c}{(1-s)(I+B)} \right] = \left[\frac{c}{s(I+B)} \right] \dots (2)$$

Taking first 2 expressions from (2):

$$\begin{aligned} \frac{(b-c)}{\eta(1-s)wh(\tau+e)} &= \frac{\varepsilon b}{e(1-s)wh\eta} \\ \Rightarrow \frac{(b-c)}{(\tau+e)} &= \frac{\varepsilon b}{e} \\ \Rightarrow e(b-c-\varepsilon b) &= \varepsilon \tau b \end{aligned}$$

$$e^* = \frac{\varepsilon \tau b}{\{b(1-\varepsilon) - c\}}$$

$$\begin{aligned} e^* &= \frac{\varepsilon \tau b}{\{b(1-\varepsilon) - c\}} \\ \Rightarrow (e^* + \tau) &= \frac{\tau(b-c)}{\{b(1-\varepsilon) - c\}} \end{aligned}$$

Putting the value of e^* in (2):

$$\frac{\{b(1-\varepsilon) - c\}}{\eta wh(1-s)\tau} - \frac{c}{(1-s)(I+B)} = \frac{c}{s(I+B)}$$

(expression 1 and 2 becomes equal)

$$\begin{aligned} \Rightarrow \frac{(I+B)\{b(1-\varepsilon) - c\} - c\eta wh\tau}{(1-s)\eta wh\tau} &= \frac{c}{s} \\ \Rightarrow s(I+B)\{b(1-\varepsilon) - c\} &= c(1-s)\eta wh\tau + s\eta wh\tau \\ \Rightarrow c\eta wh\tau &= s\{b(1-\varepsilon) - c\}\{wh - wh\eta(e + \tau) + B\} \\ \Rightarrow c\eta wh\tau &= s\{b(1-\varepsilon) - c\} \left[wh - \frac{wh\eta\tau(b-c)}{\{b(1-\varepsilon) - c\}} + B \right] \\ \Rightarrow c\eta wh\tau &= s[wh\{b(1-\varepsilon) - c\} - wh\eta\tau(b-c) + B\{b(1-\varepsilon) - c\}] \\ \Rightarrow c\eta wh\tau &= s[(wh+B)\{b(1-\varepsilon) - c\} - wh\eta\tau(b-c)] \\ \Rightarrow s &= \frac{c\eta wh\tau}{[(wh+B)\{b(1-\varepsilon) - c\} - wh\eta\tau(b-c)]} \end{aligned}$$

Putting this value of s in the constraint $[C_t \geq \bar{C}]$:

$$\begin{aligned} (1-s)\{wh - wh\eta(e + \tau) + B\} &\geq \bar{C} \\ \Rightarrow \left[\frac{(wh+B)\{b(1-\varepsilon) - c\} - wh\eta\tau b}{(wh+B)\{b(1-\varepsilon) - c\} - wh\eta\tau(b-c)} \right] \left[wh - wh\eta \cdot \frac{\tau(b-c)}{\{b(1-\varepsilon) - c\}} + B \right] &\geq \bar{C} \\ \Rightarrow \left[\frac{(wh+B)\{b(1-\varepsilon) - c\} - wh\eta\tau b}{(wh+B)\{b(1-\varepsilon) - c\} - wh\eta\tau(b-c)} \right] \left[\frac{(wh+B)\{b(1-\varepsilon) - c\} - wh\eta\tau(b-c)}{\{b(1-\varepsilon) - c\}} \right] &\geq \bar{C} \\ \Rightarrow (wh+B)\{b(1-\varepsilon) - c\} - wh\eta\tau b &\geq \bar{C}\{b(1-\varepsilon) - c\} \end{aligned}$$

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$$\Rightarrow wh\eta\tau b \leq (wh + B - \bar{C})\{b(1 - \varepsilon) - c\}$$

$$\eta^* \leq \frac{(wh + B - \bar{C})\{b(1 - \varepsilon) - c\}}{bwh\tau}$$

Putting the value of e^* and η^* into the constraint:

$$(1 - s)[wh - wh\eta(e + \tau) + B] \geq \bar{C}$$

$$\Rightarrow (1 - s) \left[wh - wh \frac{(wh + B - \bar{C})\{b(1 - \varepsilon) - c\}}{bwh\tau} \frac{\tau(b - c)}{\{b(1 - \varepsilon) - c\}} + B \right] \geq \bar{C}$$

$$\Rightarrow (1 - s) \left[(wh + B) - \frac{(wh + B - \bar{C})(b - c)}{b} \right] \geq \bar{C}$$

$$\Rightarrow (1 - s)[whb + bB - whb + whc - bB + cB + b\bar{C} - c\bar{C}] \geq b\bar{C}$$

$$\Rightarrow (1 - s)[c(wh + B) + \bar{C}(b - c)] \geq b\bar{C}$$

$$\Rightarrow s \leq \frac{c(wh + B) + \bar{C}(b - c) - b\bar{C}}{c(wh + B) + \bar{C}(b - c)}$$

$$s^* \leq \frac{c(wh + B - \bar{C})}{c(wh + B - \bar{C}) + b\bar{C}}$$