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Conditional Value-At-Risk and Currency Hedging

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This study evaluates the efficacy of the transaction exposure (value-at-risk, or VaR) for a multinational corporation (MNC) conducting business in sixteen specific currencies for the 2011-2016 time period. The approaches include traditional VaR (also called the variance-covariance approach), modified MVaR and conditional CVaR. More specifically, the maximum 1-day holding period loss is computed and compared across the three approaches to provide practical information to assist MNCs in understanding the extent to which each version of value at risk can be beneficial over time. These results also provide MNCs critical information in determining howthey should hedge this risk and which currencies to hedge. Events like Brexit and other significant events impact MNCs currency risk. By knowing what is the maximum loss conditional on a specified VaR will immensely help firms manage their currency risk much more precisely.

1. Introduction

The fixed exchange rateregime ended nearly four decades ago. Since then the floating exchange rates with all its uncertainties, volatilities and complications have challenged corporations to mitigate their currency risk exposure. Globalization and increasing interconnected nature of financial markets has warranted sophisticated risk management techniques. Different national currencies must still be exchanged to conduct global business. Most recently, in 2016 Brexit has roiled the currency markets. Accordingly, MNCs must quantify the exchange rate risk exposure and hedge their risks in order to minimize any potential loss.

There are three types of risk due to exchange rate movements: transaction, economic, and translation. For many MNCs, due to their daily operations, transaction exposure/risk are generally of an overriding magnitude. Hence, thistransaction risk is the principal focus of this study.

The keyadvantages of using value-at-risk (VaR) as a risk management tool are: (i) it places a specific dollar value on the downside risk (i.e., the maximum likely loss) an MNC will face, (ii) over a fixed time period over which risk is assessed and (iii) at a particular confidence interval. VaR assumes that data under study is normally distributed whereas modified value-at-risk (MVaR) takes into account skewness and/or excess kurtosis for any non-normal distribution data. Furthermore, we also use conditional value-at-risk assess the likelihood that a specific loss will exceed the value-at-risk. This is the most important contribution of this paper to the existing literature.

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This study estimates the specific dollar value using conditionalvalue-at-risk at 99% confidence level.The maximum 1-day holding period losses, under each of the three variations of VaR, are estimated and compared. This study will help MNCs understand the maximum one-day loss due to the currency and develop suitable hedging strategies to minimize their potential losses. By providing the conditional value-at-risk measure to hedge currency risk, this study fills the void in the current literature. The results also provide MNCs with critical information as to whether or not currency hedging is warranted in the form of a minimum-variance portfolio.

This study is unique since it is the first time all three different value-at-risk approaches are used to estimate the maximum one day loss for MNCs.This paper is organized as follows. The statement of purpose gives the background and rationale for the study followed by the literature review. Then the methodology section describes empirical tools employed in this study. The data section provides information about the data sources and finally the results of this study and discusses the conclusions and limitations.

2. Statement of Purpose

Businesses to aid in their growth expand overseas. With goldno longer being the backbone of U.S. dollar, and with the subsequent depreciation of the U.S. dollar, high volatility of exchange rates in the foreign exchange market has become the norm. Hence, successful financial managers must carefully evaluate the operating environment to measure the perceived risk (i.e., exposure risk). The increase in the level of transaction exposure that firms face can be captured by the rapidly increasing level of foreign currency transaction over the last twenty years. In 1985, approximately $ 0.14 trillion a day was the reported level of daily foreign exchange transactions. This was just twelve years following the flexible exchange rates resulting from the breakdown of the Bretton Woods System. Ten years later, by 1995,the level of foreign currency transactions reached nearly $1 trillion a day. By 2016, the reported level of daily foreign exchange transactions was over $ 5 trillion.

With the increase in exchange rate volatility, along with the rapidly rising level of foreign currency transactions resulting from the globalization, MNCs benefit by utilizing by value-at-risk as a risk management tool. Transaction risk involves the firm's operations denominated in the foreign currencies. It is indispensable that firm manage this transaction risk. By estimating and comparing the three approaches of value-at-risk (VaR), the modified approach (MVaR) and conditional approach (CVaR) for specific currencies and for a specific time period, this study provides a pragmatic method to firms seeking an efficient or optimal risk management strategy. The conditional value-at-risk measure, for seventeen currencies over five year period (January 2011 – December 2016), helps firms quantify the maximum loss above and beyond what the traditional value-at-risk provides. We use the last five year data to estimate the rolling standard deviation averages which is essential to VaR computation. Previous studies (Chiu, Lee and Hung 2005) have shown that the longer the period, less accurate are the VaR measures.

We measure the riskiness of each of the seventeen currencies individually as well the riskiness of the multiple currency portfolios aggregated as developed countries and developing countries. The results help multinational firms or similar business entities to devise alow cost risk mitigation strategy. This information also provides MNCs with important information in deciding if hedging is warranted and which currencies to hedge. Due to the paucity of this applied research in this area, this study adds to the current literature in guiding firms on how to manage their currency risk effectively.

3. Literature Review

The value at risk methodology has a variety of applications as a risk management tool. It is particularly useful as a measure of the downside risk associated with fluctuating asset values. This includes securities, commodity prices, loan portfolios, and derivatives as well as foreign currency values. It was endorsed, early on, by the U. S. Securities and Exchange Commission which requires that all large corporations report approximations of their market risk in their (public) financial statements using the value at risk model or an alternative approach. This SEC requirement is reviewed by Linsmeier and Pearson (1997) and by Theim and Ruiz-Zaiko (1998).

The use of the value at risk has gained traction, as well, from recommendations by the Basel Committee on Banking Supervision that depository institutions should report publicly estimates of their “trading risk” in the form of a simple and understandable value at risk metric. The article by Jorion (2002) shows that the reported values at risk in U.S. commercial banks are correlated with the volatility in trading revenues and hence can be used as a proxy for risk. In a similar way, value at risk has become is an increasingly popular way for financial institutions to measure the risk of holding assets in multiple currencies. The article by Cayton, Mapa and Lising (2010) is a recent example and so are studies by Escanciano, Olmo (2012), Khazeh et al (2014), Khazeh et al (2016).

In fact, the value at risk approach to quantifying risk actually includes a number of specific methodologies including the variance-covariance (or delta-normal) approach, historical simulation and Monte-Carlo simulation. As noted above, the value at risk model has obvious applications for quantifying the risks associated with investments in various money and capital market securities or portfolios of these securities (denominated in either the domestic currency or a foreign currency).

Lu, Kawai and Maekawa (2010) use the basic value at risk framework to estimate the risks associated with five different exchange rates (i.e., all vis-à-vis the U.S. dollar) due to the revaluation of the Chinese yuan in July 2005. Mabrouk and Chaker(2011) and Rejeb, Salha and Rejeb (2012), Khazeh and Winder (2010) also employ value at risk methodology to estimate the exchange rate risk associated with multiple currencies and currency portfolios. Mohammadi and Akhtekhane (2012) estimate the risk associated with the U.S. dollar/rial exchange rate using value at risk.

Despite the obvious advantages of the value at risk approach to risk management, including its relative simplicity and intuitive appeal, the approach has its weaknesses. In particular, the variance-covariance approach (also called the "parametric" or "delta normal" approach) to measuring value at risk assumes that the variable or variables under consideration are normally distributed. However, if the probability distributions are non-normal, and in particular if the probability distributions have "fat tails," standard value at risk calculations may give unreliable estimates of the downsides risks. Under these conditions, alternative approaches to estimating value at risk, including what is called "extreme value theory," may be superior. Articles relating to fat tails, extreme value theory, and how to quantify value at risk when the assumption of normality is violated include Cayton and Mapa (2012). Articles using "copula models" (a branch of extreme value theory) to assess value at risk include Jaworski (2008) and Huang, Chien and Wang (2011).

The development of "conditional value at risk" (CVAR) methodology represents an attempt to modify the value at risk model to apply to cases where the loss is expected to exceed that forecast by the standard VAR model. Specifically, CVaR attempts to estimate exactly how large the loss is likely to be at a particular confidence level, given (assuming) that it will be larger than the standard VAR. The article by Rockafellar and Uryasev(2000) was the first to introduce the term of conditional value-at-risk. Chiu, Lee and Hung (2005) note that the accuracy of the value at risk methodology may diminish as the time horizon (i.e., the holding period) lengthens. Yamai and Yoshiba (2002) show that for large sample sizes increase the accuracy of CVaR estimation especially for fat-tailed distributions.

The motivation for this research is to help multinational firms minimize their cost of reducing currency risk using hedging. Prior studies have focused only on VaR and MVaR maximum one-day loss, this paper uses conditional value-at-risk to capture losses beyond what a traditional VaR model will capture. By doing so, firms can be better prepared to minimize any potential currency risk exposure. This is the first paper to compare the three different value-at-risk measures for maximum one-day currency loss.

4. Methodology

In recent years, the traditional approach to value-at- risk (VaR), and to some degree its variations (MVaR), have been commonly used among the most popular approaches inmeasuring transaction risk. It is used to estimate the prospective maximum 1-day, 1-week, 1-month, and even longer period losses of MNCs ongoing cash flows that are denominated in other currency/currencies.

In this study, the results for the maximum 1-day holding period loss for an individual currency "i" or for a portfolio of currencies "p" can be estimated using equation 1:

$$E(e_t) - (Z)x\,(\sigma_{i \text{ or } p}) \qquad (1)$$

where:

The expected percentage change in the currency's value for the relevant period = $E(e_t)$

The z-score corresponding to the desired confidence level used (i.e., 95%, 97.5%, 99%) in the case of 95% = -1.65, for 97.5% = -1.96, and for 99% = -2.326.

The standard deviation of the percentage change in the currency's or portfolio of currencies value over previous period =$\sigma_{i \text{ or } p}$

The results for the modified maximum 1-day holding period loss for an individual currency "i" or for a portfolio of currencies "p" can be estimated using equation 2:

$$E(e_t) - (ModZ)x\ (\sigma_{i \text{ or } p}) \qquad (2)$$

where:

The expected percentage change in the currency's value for the relevant period = $E(e_t)$

The modified Z-score corresponding to the desired confidence level used (i.e., 95%, 97.5%, 99%) in the case of 95% = -1.65, for 97.5% = -1.96, and for 99% = -2.326. In this case *Mod Z*is computed as

$$ModZ = (z + \frac{Skew(z^2-1)}{3!} + \frac{Kurt(z^3-3z)}{4!} + \frac{Skew^2(2z^3-5z)}{36})$$

where z is the normal z-score corresponding to the desired confidence level

Skew is the skewness of the population

Kurt is the excess kurtosis or absolute kurtosis -3

The standard deviation of the percentage change in the currency's or portfolio of currencies value over previous period =$\sigma_{i \text{ or } p}$

The values of the standard deviations for an individual exchange rate, as well as for a portfolio of currencies, are shown in Equations 3 and 4 (respectively), below. Additionally, Equation5 constitutes an application of Equation4 for a two-currency portfolio.

$$\sigma = \sqrt{\frac{\sum(x-\overline{x})^2}{N}} \qquad (3)$$

$where$:

$\sigma = the\ standard\ deviation$

$x = each\ value\ in\ the\ population$

$\overline{x} = the\ mean\ of\ the\ values$

$N = the\ number\ of\ values\ (the\ population)$

$$\sigma_p = \sqrt{\sigma_p^2} \tag{4}$$

$$\sigma_p^2 = \sum_i w_i^2 \sigma_i^2 + \sum_i \sum_{j \neq i} w_i w_j \sigma_i \sigma_j \rho_{ij}, \tag{4A}$$

where:

σ_p *= standard deviation of weekly percentage changes in currency portfolio*

σ^2_p *= variance of weekly percentage changes in currency portfolio*

w_i *= proportion of total portfolio value denominated in currency i*

w_j *= proportion of total portfolio value denominated in currency j*

σ_i *= standard deviation of weekly percentage changes in currency i*

σ_j *= standard deviation of weekly percentage changes in currency j*

ρ_{ij} *= correlation coefficient of weekly percentage changes between currencies i and j*

$$\sigma_P = \sqrt{W_X^2 \sigma_X^2 + W_Y^2 \sigma_Y^2 + 2 W_X W_Y \sigma_X \sigma_Y CORR_{XY}} \tag{5}$$

where:

σ_p *= the standard deviation of the two-currency portfolio*
W_X *= proportion of the total portfolio in currency X*
W_Y *= proportion of the total portfolio in currency Y*
σ_X *= the standard deviation in the weekly percentage changes in currency X*
σ_Y *= the standard deviation in the weekly percentage changes in currency Y*
$CORR_{XY} = \rho_{XY}$ *= the correlation coefficient of the weekly percentage changes between currencies X and Y*

Conditional Value at Risk (CVaR) is a probability weighted average of the possible losses conditional on the loss being equal to or exceeding the specified VaR.

To compute CVaR, if the periodic returns are normally distributed, then:

$$CVaR_Z = \frac{e(-VaR_Z\text{^}2)/2)}{(\alpha*(2*\pi)\text{^}0.5} \tag{6}$$

where:

$CVaR_Z$ = adjusted Z-score
VaR_z = z score of confidence level used
α = 1 – confidence level

The CVaR percentage is calculated using the following equation:

$$CVaR = \mu + CVaR_Z\sigma_i (7)$$

where:
μ= mean return
σ_i = standard deviation of the percentage change in the currency's value
$CVaR_Z$ = Adjusted Z-score

In this study, we assume the periodic returns are normally distributed and compute the CVaR values at 99% confidence level. The CVaR values gives us the maximum possible loss beyond the traditional VaR at 99% confidence interval. It can be called as mean excess loss or tail VaR and is considered a consistent measure of risk than VaR.

5. Data

The countries and their corresponding currencies included in this analysis are obtained from the U.S. Federal Reserve System currency database. The nine developed country currencies are: the Australian dollar, the British pound, the Canadian dollar, the Euro, the Hongkong dollar, the Japanese yen,the Norwegian Kroner, the Singapore dollar and the Swiss franc.The Brazilian real, Indian rupee, Korean won, Mexican peso, South African rand, Taiwanese dollar and Thai baht constitute the developing country currencies. The time period for the basis of this study is the latest daily currency rates available for January 3, 2011 – December31, 2016. We selected this time period as it includes some of the significant events such as the Swiss franc decoupling from the Euro and the dramatic impact of Brexit on the British pound. This time period includes in excess of one thousand, four hundred and seventy daily observations on the relevant spot exchange rates.We do not use longer data set as previous studies have shown that the accuracy of the value at risk methodology may diminish as the time horizon (i.e., the holding period) lengthens. Besides, our study's data period is in line with previously published papers in the foreign exchange markets. In order to compute the standard deviation of the individual currency (Equation 3), the rolling average method is utilized.For January 3, 2011, the computation of the standard deviation of the returns for the first 257 observations starts from January 3, 2011 to December 31, 2011. For the next day (January 4, 2011), the computation of the standard deviation of the returns starts from January, 4, 2011 through January 3, 2012 and so on.

6. Results

Table 1 present the descriptive statistics of developed country currencies daily percent changes is given. Of the nine currencies, the Swiss franc followed by British pound and Norwegian kroner have the largest variability during this time period.

Table 1: Daily Percent Changes for Developed Countries' Currencies (2011-2016)

	AUS	EURO	UK	CAD	HKD	JPY	NOK	SGD	CHF
S_0 mean	-0.0168%	-0.0119%	-0.0132%	-0.0193%	0.0001%	-0.0152%	-0.0212%	-0.0049%	-0.0006%
S_1 mean	-0.0165%	-0.0120%	-0.0133%	-0.0191%	0.0001%	-0.0160%	-0.0215%	-0.0054%	0.0000%
$(S_1 - S_0)/S_0$	-0.0176%	0.0088%	0.0117%	-0.0112%	0.0072%	0.0504%	0.0137%	0.0990%	-1.0821%
DailyChange Min	-4.3589%	-2.1718%	-7.8446%	-3.3117%	-0.3340%	-3.2876%	-4.6676%	-2.6589%	-8.5069%
DailyChange Max	3.2663%	3.1117%	2.8211%	2.9425%	0.2849%	3.5596%	3.2180%	2.4136%	13.9082%
Count	1471	1471	1471	1471	1471	1471	1471	1471	1471

Table 2 provides the descriptive statistics for developing country currencies daily percent changes. Among the eight developing countries, the South African rand, Brazilian real and Indian rupee respectively have the largest variation during the 2011-2016 time period.

Table 2: Daily Percent Changes for Developing Countries' Currencies

	BRL	INR	KRW	MXN	ZAR	TWD	THB
S_0 Mean	-0.0411%	-0.0257%	0.0000%	-0.0299%	-0.0435%	-0.0050%	-0.0098%
S_1 Mean	-0.0431%	-0.0256%	-0.0007%	-0.0323%	-0.0454%	-0.0053%	-0.0102%
$(S_1 - S_0)/S_0$	0.0500%	-0.0032%	15.4931%	0.0795%	0.0442%	0.0604%	0.0425%
DailyChange Min	-4.4275%	-3.7209%	-2.7721%	-6.7739%	-5.0163%	-1.6480%	-1.4395%
Daily Change Max	5.4420%	3.8274%	3.0074%	3.1537%	6.4574%	1.6439%	1.9981%
Count	1471	1471	1471	1471	1471	1471	1471

Table 3: Correlation Coefficients for Developed Country Currencies based on Daily Percentage Changes

	AUS	EURO	UK	CAD	HKD	JPY	NOK	SGD	CHF
AUS	1.0000	0.4911	0.4829	0.6793	0.2705	0.1877	0.5904	0.7262	0.3185
EURO	0.4911	1.0000	0.5686	0.4456	0.1584	0.2886	0.7076	0.5954	0.5900
UK	0.4829	0.5686	1.0000	0.4732	0.1737	0.1125	0.5497	0.5154	0.3581
CAD	0.6793	0.4456	0.4732	1.0000	0.2393	0.1020	0.5760	0.6208	0.2802
HKD	0.2705	0.1584	0.1737	0.2393	1.0000	-0.0324	0.1921	0.3031	0.0817
JPY	0.1877	0.2886	0.1125	0.1020	-0.0324	1.0000	0.1705	0.2826	0.3179
NOK	0.5904	0.7076	0.5497	0.5760	0.1921	0.1705	1.0000	0.6077	0.4426
SGD	0.7262	0.5954	0.5154	0.6208	0.3031	0.2826	0.6077	1.0000	0.4142
CHF	0.3185	0.5900	0.3581	0.2802	0.0817	0.3179	0.4426	0.4142	1.0000

Table 3 shows correlation coefficients among the developed country currencies. The three highest correlation are among Australian dollar-Singapore dollar at 0.73, Euro-Norwegian kroner at 0.71 and Australian dollar-Canadian dollar at 0.68. The three least correlated currency pairs in the developed countries are Japanese yen-Hongkong dollar at -0.03, Swiss franc - Hongkong dollar at 0.08 and Japanese yen-Canadian dollar at -0.10.

Table 4: Correlation Coefficients for Developing Country Currencies based on Daily Percentage Changes

	BRL	INR	KRW	MXN	ZAR	TWD	THB
BRL	1	0.2974	0.3886	0.5722	0.5495	0.3830	0.3548
INR	0.2974	1	0.3628	0.4142	0.3911	0.3476	0.4202
KRW	0.3886	0.3628	1	0.5007	0.5091	0.6710	0.4785
MXN	0.5722	0.4142	0.5007	1	0.6750	0.4491	0.4107
ZAR	0.5495	0.3911	0.5091	0.6750	1	0.4870	0.4564
TWD	0.3830	0.3476	0.6710	0.4491	0.4870	1	0.4445
THB	0.3548	0.4202	0.4785	0.4107	0.4564	0.4445	1

Table 4 provides correlation coefficients among the developed country currencies. The highly correlated currency pairs are the Mexican peso-South African rand at 0.68, Korean won-Taiwan dollar at 0.67 and Mexican peso-Brazilian real at 0.57. The least correlated developing country currency pairs are Chinese yuan-Indian rupee at 0.131, Chinese yuan-Brazilian real at 0.136 and Chinese yuan-Mexican peso at 0.148.

Table 5: VaR, MVaR, CVaR Maximum Individual Developed Country Currency Daily Loss

	$(S_1 - S_0)/S_0$	**VaR**(@99%)	**MVaR**(@99%)	**CVaR** (@99%)
AUS	-0.0176%	-1.6742%	-1.4634%	-1.9155%
EURO	0.0088%	-1.3786%	-1.2335%	-1.5776%
UK	0.0117%	-1.3147%	-4.4623%	-1.5044%
CAD	-0.0112%	-1.2171%	-1.2122%	-1.3915%
HKD	0.0072%	-0.0742%	-0.2209%	-0.0850%
JPY	0.0504%	-1.4453%	-1.5518%	-1.6534%
NOK	0.0137%	-1.7661%	-1.4838%	-2.0202%
CHF	-1.0821%	-1.7753%	-21.9578%	-2.0337%
SGD	0.0990%	-0.8895%	-1.0968%	-1.0182%

Table 5 shows the comparisons of these maximum one day loss for each of the developed country currency using the three approachesVaR,MVaR and CVaR. The first column $(S_1 - S_0)/S_0$is the actual loss based on the average daily returns over the 2011-2016 time period. The modified value-at-risk values for each currency gives better and more accurate value of the loss than the traditional value-at-risk as observed in past studies. It is evident the conditional CVaR captures the maximum one-day loss for each developed currency beyond the traditional VaRat the 99% confidence level.

The reasons for the modified VaR losses being greater than condition VaR for the British pound can be attributed to the impact of Brexit and resulting volatility on the British pound relative to the U.S. dollar. Similarly, in January 15, 2015 the Swiss central bank decision to unpeg the Swiss franc from the Euro and allowed it to depreciate led to significant decline in the Swiss franc exchange rate relative to the Euro as well as the

U.S. dollar. Previously, on September 6, 2011 the Swiss franc was pegged to the Euro in order to protect its currency from the adverse impact of the European debt crisis. The conditional VaRis a probability weighted average of the possible losses conditional on the loss being equal to or exceeding the specified VaR. Since we are measuring VaR and CVaR at the same confidence level of 99%, they both are measuring different parts of the distribution. The CVaR captures any loss that the traditional VaR fails to capture due to the non-normality of the data, specifically the fat-tailed distributions.

Table 6: CVaR Maximum One-Day Loss of $1million Dollar Currency

AUS	-$19,155
EURO	-$15,776
UK	-$15,044
CAD	-$13,915
HKD	-$850
JPY	-$16,534
NOK	-$20,202
CHF	-$20,337
SGD	-$10,182

Table 7: VaR, MVaR, CVaR Maximum Individual Developing Country Currency Daily Loss

	$(S_1 - S_0)/S_0$	**VaR** (@99%)	**MVaR** (@99%)	**CVaR** (@99%)
BRL	0.0500%	-2.2693%	-2.3184%	-2.5934%
INR	-0.0032%	-1.2929%	-1.8092%	-1.4774%
KRW	15.4931%	-1.2890%	-1.2355%	-1.4766%
MXN	0.0795%	-1.7235%	-2.1909%	-1.9699%
ZAR	0.0442%	-2.3699%	-2.2969%	-2.7084%
TWD	0.0604%	-0.7191%	-0.8579%	-0.8231%
THB	0.0425%	-0.7329%	-0.7466%	-0.8382%

Similarly, table 7 shows that conditional VaR consistently provides a much large maximum one-day loss for each developing country currency over the traditional VaR at the 99% confidence level.

Table 8: CVaR Maximum One-Day Loss of $1million Dollar Currency

BRL	-$25,934
INR	-$14,774
KRW	-$14,766
MXN	-$19,699
ZAR	-$27,084
TWD	-$8,231
THB	-$8,382

Since the time period chosen is five year period and previous studies have shown that longer periods weakens the value-at-risk estimates. Based on the results, we can identify the most appropriate hedging strategy for MNCs to hedge their foreign exchange risk.

Table 9: Minimum Variance Portfolio Weights for Developed Countries

AUS	EURO	UK	CAD	HKD	JPY	NOK	SGD	CHF
15.75%	13.80%	11.15%	10.42%	0.22%	7.58%	17.59%	8.73%	14.76%
Portfolio Std. Dev				0.4680%				

The minimum-variance portfolio for developed countries suggests that hedging the following currencies Euro, British pound and Australian dollar will yield the developed country currency portfolio with the least amount of risk. The rationale for hedging using the Euro instead of Swiss franc or Norwegian kroner is the relatively high correlation coefficient between the currency pairs of Euro-Swiss franc and Euro-Norwegian kroner. Besides, hedging with a widely used currency will be minimize the costs. Hedging in currencies is expensive for firms. Our results show that a multinational firm need not hedge all the nine currencies that it has exposure, rather it can hedge three currencies and achieve equivalent risk reduction in any exchange rate risk exposure.

Table 9A: Minimum Variance Portfolio Weights for Australian Dollar, British Pound, and Euro

AUS	EURO	UK
38.76%	31.58%	29.66%
Portfolio Std. Dev		0.4774%

We computed the mini-variance portfolio using these three currencies: Australian dollar, Euro and the British pound. The optimal weights suggest that a multinational firm can reduce its currency risk by hedging these three currencies. Hedging currency risk is expensive. Our results show that the firm can minimize its maximum one-day loss by hedging three currencies out of a total of nine currencies. This is 67% reduction in hedging costs and will be substantial to the bottom line of a multinational firm.

Table 10: Minimum Variance Portfolio Weights for Developing Countries

BRL	INR	KRW	MXN	ZAR	TWD	THB
22.25%	9.82%	11.98%	18.19%	25.77%	6.17%	5.82%
Portfolio Std. Dev			0.6017%			

In case of the developing countries' currencies, the minimum variance portfolio suggests hedging in South Africa rand, Brazilian real and the Korean won. These three currencies have relatively low coefficients of correlation among them and would give a reasonable risk reduction. By choosing only three out of the seven currencies to hedge, multinational firms can reduce their foreign currency risk significantly. Our study provides a guide as how firms reduce their currency risk at a relatively lower cost of hedging.

Table 10A: Minimum Variance Portfolio Weights for South Africa Rand, Brazilian Real, and Korean Won

BRL	INR	KRW
51.89%	22.81%	25.30%
Portfolio Std. Dev		0.6103%

In order to obtain a much more compact optimal portfolio, we computed the mini-variance portfolio consisting of the Brazilian real, Indian rupee and the Korean won. We find that the portfolio standard deviation is not significantly different between these three currencies and the seven currency developing countries' portfolio. Our results indicate that a multinational firm will be able mitigate the currency risk in developing countries by hedging these three currencies instead of the seven currencies.

6. Conclusions

In this study, we have shown how multinational firms can reduce their tail risks by using the conditional value-at-risk measure to capture maximum one-day loss. We show that the conditional value-at-risk is a better predictor of the maximum one-day loss over the traditional value-at-risk measure. The minimum-variance portfolio of currencies results highlight that firms can reduce their hedging costs by selectively hedging a few currencies instead of all currencies in the portfolio. Our results show that a multinational firm will reduce its developed countries' currency risk by hedging the Australian dollar, Euro and the British pound out of the nine developed countries in which it operates. Similarly, the results imply by hedging Brazilian real, Indian rupee and the Korean won, a multinational firm can reduce its currency risk in seven developing countries. The study is significant as it provides a clear, sensible strategy to reduce currency risk. Though a firm might operate and have currency risk in sixteen different currencies, it needs to hedge only six currencies to reduce its transaction risk. The savings obtained by hedging six currencies instead of sixteen is substantial. The cost savings for the firm is to 62.5% Firms can hedge either using forward contracts or currency futures. We have repeated this study using 5-day and 10-day returns and the results are still the same.

As there are no prior studies that contrast the three different value-at-risk measures, our paper contributes the literature with results. Our study is the first of its kind to provide a clear guideline on how multinational firms can minimize their currency risk exposure and quantify which currencies to hedge and in what proportion. This study is quite appropriate given the recent volatility in the currency markets due to events such as Brexit and Swiss franc decoupling from the Euro. Most multinational firms realize their revenues overseas in countries listed in this study. It is indispensable that firms have a prudent hedging strategy to reduce their currency risk exposure. The limitation of this study is that we assume that typical multinational firm operates in the sixteen currencies considered in the study. Some multinational firms might have currency risk exposure in currencies that are not part of this study. Also, this research covers five years of data from 2011-2016. The risk exposure and consequently the optimal portfolio weights might change if the period changes. So, it will be prudent for firms to calculate the risk exposure and optimal portfolio for the most recent period to manage their currency risk.

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