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# Skewness on Equity Portfolio Selection: Evidence from the US Stock Market

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*This paper uses empirical test on simple portfolios with two stocks only, then extending to portfolios selected based on various criteria such as industry sectors, correlation coefficient, random pick etc., to test on the effect of skewness to the risk of the selected portfolio. From the experiment results generated by selected portfolios with two stocks only, we find that the portfolio with negative skewness is the weakest in risk reduction. Extending to random portfolio scheme, in US stock markets, we run regressions on the portfolio skewness to the risk reduction of the portfolio and discovered the opposite facts, the two variables are actually negatively related. The results indicate that the more positive skewed stocks are chosen in a portfolio, by controlling the industry sector and the size of the firm, the smaller the portfolio risk is reduced.*

**JEL Codes:** G11,G12, and G19

## 1. Introduction

The debate on skewness in stock returns has also apparently drawn attention on equity portfolio selection since individual stocks are the major elements in a portfolio. H. Markowitz made a speech in Baruch College when he earned the Nobel Prize in 1990 for introducing the modern portfolio theory. During the speech, he admitted that if one can overcome computational difficulties, there is “more to be done” in incorporating higher moments (i.e. third and fourth moments such as skewness and kurtosis) into portfolio modeling. In his modern portfolio theory, Markowitz adopts the use of mean-variance evaluation to trade off risk and return. However, a recent study by DeMiguel, Garlappi, and Uppal (2009) shows that nowadays a lot of mean-variance (MV) portfolio models designed to reduce the effect of estimation error cannot efficiently increase return or decrease risk than any other simpler portfolios. On the other hand, some like Roman, Darby-Dowman, and Mitra (2007) try to ameliorate the MV model by introducing more risk factors. Unsatisfied at all the formal approaches, some start to look at higher moments to include into the MV model. And based on the papers discussing individual stock returns, many believed that including skewness in the MV model might make a better portfolio.

Skew, or skewness can be mathematically defined as the averaged cubed deviation from the mean divided by the standard deviation cubed. If the result of the computation is greater than zero, the distribution is positively skewed. If it's less than zero, it's negatively skewed. If the result is equal to zero then it means that the distribution it's symmetric. For interpretation and analysis, skewness is always used to focus on downside risk. Negatively skewed distributions have a long left tail, which for investors can mean a greater chance of extremely negative outcomes. Positive skew would mean frequent small negative outcomes.

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Study by Arditti in 1967 shows that investors prefer positive skewness rather than negative since positive skewness means a fat tail to the right and even though this means more frequent small negative returns, it greatly decreases possibilities of extreme negative returns. On behavioral finance side, stocks with a positive skewness return distributions seem more preferable especially to risk-averse investors as the distribution skew to the right and extremely negative values will rarely occur. However, this assumption is yet lack of empirical support and the exact effect of skewness to portfolio return and risk is still under discussion. It is still not clear if one should include as many positively skewed stocks as possible or if there should be a combination of both positive and negative in order to optimize the portfolio. The discoveries of the effect of skewed stocks in a portfolio shed lights to those who want to improve the original portfolio theory by including higher moments. It inspired the mean-variance-skewness model seekers by optimizing the skewness to an either positive of moderate level. Many methods have incorporated this optimization concept in their model.

This paper will first look at different methods used in formulating a mean-variance-skewness model on portfolios and then use empirical data to study the behavior of portfolio skewness on portfolio risk reduction. The rest of papers are organized as followings. In section 2, we discuss literature review on portfolio selections and skewness. In section 3, we examine the methodology and the model setup. In section 4, we present the findings. Section 5 concludes the paper.

## 2. Literature Review

In the past fifty years, many papers are devoted to optimize portfolios by incorporating the skewness factor. As Arditti argues in 1967[1], consumers generally prefer positive skewness to lower the possibility of having extremely negative returns. Based on this assumption, which is not fully theoretically proven or thoroughly empirically tested, many studies are carried out aiming to fuse skewness as an important factor into portfolio selection. Therefore, many aimed to maximize, or say, achieve a positive skewness, when optimizing a portfolio.

### 2.1 Portfolio Selection

There are several papers published that are focusing on such in portfolio selection. One paper wrote by Walter Briec, Kristiaan Kerstens and Ignace Van de Woestyne did an outstanding job by collecting all the most palpable methods presented in the last 20 years. Their paper, "Portfolio Selection with Skewness: A Comparison and A Generalized Two Fund Separation Result"[2], enumerated many former approaches. In 2002, Wang Xia determined MVS portfolios via a multi-objective programming approach. In 2004, Athayde and Flores determined analytical solutions characterizing the MVS portfolio frontier by minimizing the variance for a given mean and skewness while assuming a risk-free asset and shorting.

The paper also mentioned other primal research lines as it wrote: "For example, Li, Qin, and Kar (2010) develop a fuzzy MVS model as well as some variations. As another instance, Konno, Shirakawa, and Yamazaki (1993) formulate a general portfolio optimization problem maximizing skewness subject to fixed expected return and variance constraints, whereby both the quadratic and cubic terms are linearly approximated to yield a mean-absolute deviation skewness model. Boyle and Ding (2005) pick up from there and check the effect of only using a linear approximation for the cubic term. Note that a lot of these contributions tend to solve the MVS portfolio problem by privileging one or two of the objectives at the cost of the other(s). Starting from specifications of the indirect MVS utility function, dual approaches search for optimal portfolios via preference parameters reflecting attitudes towards risk and skewness. Jondeau and Rockinger (2006) and Harvey, Liechty, Liechty, and Müller (2010) are recent examples of such utility-based studies." [3]

It seems that all the methods mentioned above does not reach to a consensus and so far there has not been any work that thoroughly evaluate and compare all the methods to decide which one is better, or that some ones are intrinsically equivalent. This paper by Briec, Kerstens and Van de Woestyne published on LEM (Lille Economics and Management) in October 2011 claims that "In this context, this contribution is -to the best of our knowledge- the first attempt to develop a comparison between two primal MVS portfolio optimization approaches."

The paper by Briecm, Kerstens, and Van de Woestyne numerated examples of researches based on the model introduced by Lai (1991): "trade-offs among lower partial moments (see Chen and Shia (2007)), trade-off between return and Value-at-Risk (see Chen (2008)), index tracking (see Wu, Chou, Yang, and Ong (2007))"[4]. Also, Leung, Daouk and Chen (2001) provided a slightly generalized version of the PGP model and shifted the model from two-dimension to a three-dimensional Polynomial Goal Programming problem where the objective function L follows the computational form of Minkowsky distance[5]. Some other scholars such as Davies, Kat, and Lu (2009) also included kurtosis, the fourth moment, into the PGP model. And nowadays, Goal Programing (GP) and Multiple Criteria Decision Analysis (MCDA) are widely adopted in financial analysis. Surveys by Kosmidou and Zopounidis (2004)[6] showed that "Discrete evaluation methods are used to assist decision-making on diverse financial problems ranging from bankruptcy and credit risk assessment over investment appraisal to portfolio selection and management, among others." The popularity of Lai's paper, commented by Briecm, Kerstens, and Van de Woestyne can be interpreted as "one exponent of the widespread development of goal programming models for portfolio analysis in the operational research literature (overviews of alternative MCDA models in portfolio analysis are found in Azmi and Tamiz (2010), Spronk, Steuer, and Zopounidis (2005) or Steuer, Qi, and Hirschberger (2008))."[7]

The paper then points out another method that is quite different from the PGP model: the Shortage Function. On the opposite, the shortage function is a theoretical approach initially brought up by Cantaluppi and Hug (2000) to "directly measure portfolio performance by reference to its maximum potential on the (ex-ante or ex-post) portfolio frontier." The shortage function assumes that portfolios have risk-free assets and short selling. The traditional mean-variance function is now modified to a three-dimensional MVS space with mean, variance and also skewness. In the paper, the definition of the shortage function is illustrated as the following:

Let $g = (g_M, g_V, g_S) \in R_+ \times\sim R_- \times\sim R_+$ and $g \neq 0$. The shortage function $S_g$ in the direction of vector g is the function $S_g : R^3 \to R_+ \cup \{-\infty,+\infty\}$, with

$$\mathcal{S}_g(y) = \sup_{\delta\in\mathbb{R}_+} \{\delta\,;\, y + \delta g \in \mathcal{DR}\}.$$

In the MVS space, from a certain point to the direction of vector g, the shortage function aim to help improve the portfolio return and skewness and in the meantime reduce variance. The direction vector subjects to the investor's preference, which means that whether to prefer some two among MVS to the other one. For example, if one wishes to increase return without hurting the variance and skewness (theoretically speaking), then the direction vector can be (1,0,0). Take another example, if one wishes to minimize the portfolio risk and increase the skewness level without decreasing the portfolio return, then the vector can be (0, -1, 1). In the paper by Kerstens, Mounir, and Van de Woestyne (2011)[8], the entire process of solving the shortage function is meticulously illustrated.

Using the shortage function model is a "theoretical advantage" that "established firm link between the shortage function and the mixed risk aversion utility functions."[9] The main practical advantage of the shortage function that is pointed out by the author is "its capability of providing geometrical representations of portfolio frontiers under a wide variety of restrictions on portfolio weights."[10]

The paper respectively introduced both the PGP and the shortage function methods and then used graphical representations to analyze the similarity and individuality of the two methods. At last the paper discussed flaws on the PGP method and argued that these flaws can actually be improved by the shortage function method. Both methods are considered very helpful to this research project and they all provide insightful knowledge on the current discussion of this field.

### 2.2 On Skewness Preference

Only recently have emerged more and more empirical tests conducted to find the optimal skewness level preferred by investors. Before, many assumed that a positive skewness is more favored by especially risk averse investor. A recent paper by W. Henry Chiu used empirical tests to help prove such assumption[11]. The paper establishes a "skewness-comparability condition on probability distributions that is necessary and sufficient for any decision maker's preferences over the distributions to depend on their means, variances and, third moments only."[12] The paper poses one interesting study by Golec and Tamarkin[13] (1998) and Garret and Sobel[14] (1999) on the data from horse race betting and from state lotteries, that gamblers are not necessarily risk-lovers but skewness-lovers. The paper studied previous papers on skewness preference that are either assuming a cubic utility function or a cubic Taylor approximation of the EU (i.e, the utility function is approximated by a Taylor series truncated to three terms before taking expectations). These approaches are considered by Chiu to be rather limited. In the text, he listed and analyzed shortcomings of previous studies and their limitations. For example, he points out that one of the methodologies that use Taylor series approximation can be a reasonable approximation only for small risks.

Chiu instead cuts in from skewness comparability, which is first introduced by Van Zwet[15] (1964), then fully defined by Oja[16] (1981). Van Zwet (1964) defines that a distribution F is more skewed to the right than G if $R(x) \equiv F^{-1}(G(x))$ is convex and it has become widely accepted that a good skewness measure should preserve the skewness ordering so defined. Then he defines skewness comparability as the following:

*(i) Distributions F and G are strongly skewness comparable if $F^{-1}(G(x))$ is convex or concave.*
*(ii) F is more skewed to the right than G in the sense of Van Zwet if $F^{-1}(G(x))$ is convex*

Along using more definitions and theorems, the paper constructed the skewness comparability of the widely used Bernoulli distributions and examines its implications. Then the paper concludes with discussions on the comparison with the existing approach to modeling skewness preference, the implications for asset pricing and the decision to gamble, and the effects of progressive tax reforms on risk-taking.

This paper is one of the most thorough and orthodox empirical analysis conducted to test the true skewness preference and it proves that investors should indeed favor positive skewness. However, despite all the efforts made on skewness analysis, many still argue that these studies are not comprehensive enough and special cases may not apply. This following literature interestingly poses an assertion that skewness does not appear under some circumstances.

### 2.3 Counter Arguments on Skewness Persistency Over Time

Many recent studies show that the market return distribution is constantly positively skewed. However, the studiy shown by J. Clay Singleton and John Wingender (1986) indicates that the skewness of individual stocks and portfolios of stocks does not persist across different time periods. Positively- skewed equity portfolios in one period are not likely to be positively skewed in the next time period. Past positively skewed returns do not predict future positively skewed returns. The most important finding in this paper is that while skewness occurs with almost constant frequency in cross sections, neither individual securities nor portfolios remain skewed over time. These results actually dispute previous evidence and are inconsistent with investment policies that encourage aggressive investors to concentrate in skewed equities.

In fact, Beedles and Simkowitz[17] (1980) have done a research on the persistency on skewness by comparing the frequency of securities with skewed returns in different cross sections. They concluded that, “regardless of how skewness is measured, securities have displayed a persistent propensity to positive asymmetry during the last three decades." However, the study by J. Clay Singleton and John Wingender shows that even thought skewness is persistent in cross section data, it is not when using time series.

## 3. The Methodology and Model

Our data pool is gathered from prominent research institution on the monthly return of all the US market trading stock in one year, the year of 2011. Note that we only picked the data on one year, the year of 2011 to avoid the persistence issue at the beginning. To build on this argument, this paper thus presumes that the assertions in the literature are valid and that time series data is not efficient or may be misleading in skewness study. The literature by J. Clay Singleton and John Wingender shows that even though skewness is persistent in cross section data, it is not when using time series.

The entire pool of equities is on over 7000 stocks. We then screened off the companies that does not have continuous data or have incomplete information of the year. This helps us come up with 5680 companies within over seventy industry sectors[18]. Then we ran the descriptive on this data set of how many companies consist of each industry sector. And we selected the industries that contain more than 40 companies. This gives us a total of 5007 companies and this is our final data pool.

Portfolios are constructed by equal weight. This process will be illustrated in the following sessions. A general sense is that the portfolio return equals to the sum of the return times the weight of each element in the portfolio. A sample of a yearly return of a portfolio is as following. 11 different stocks construct the portfolio and each stock takes an equal weight. Therefore the portfolio return is just equal to $R_1/11+R_2/11+\ldots+R_{11}/11$.

After examining the empirical studies on how the individual stocks' skewness level affect the portfolio return and risk, we decide to use a different method to try to empirically test the effect of skewness on portfolio risk and return and try to grasp some sense in stock selections based on skewness. In our selected data set, which contains more than five thousand stocks with their monthly return on the year of 2011, we screened out the stocks that have significant skewness level and also the ones with skewness that is very close to 0. We then assume that $A_1$ and $A_2$ will have the same direction of effect on the two-asset portfolio and $A_1$'s effect will be more significant. This selection supports that moderate skewness will have no effect or little effect to the portfolio, but only to help us observe the effect easier. This assumption works when the two assets have the same sign of skewness, i.e. they have both positive skewness levels or they both have negative skewness. This experiment does also include the possibility when the two stocks have the opposite sign, where one has positive skewness and the other has negative skewness. Intuitively, the result of this combination is ambiguous as one positive skewed and one negative skewed asset may shift the portfolio skewness somewhere between these two polar and through this test, we will try to discover the behavior of this combination as well.

From these selected stocks, we calculate a correlation matrix that describes the correlation between every two of the stocks. By controlling the stock pairs that have correlation coefficient from 0.6 to 0.8 and -0.6 to -0.8, we test the skewness effect. These stock pairs each form a portfolio with different skewness level. In order to see the effect of individual skewness on the risk and return of its two-asset portfolio, we control for the level of correlation coefficient, is set as a specific interval rather than being categorized to different levels and analyzed respectively. In short, we will only pick the stock pairs that have quite significant correlation coefficient that is 0.6 and 0.8.

We categorize these selected stock pairs into four different groups by the combination of its individual stock skewness. To make it clear, suppose we have 1000 stock pairs, which are also known as our two-asset portfolios and two individual stocks, A and B, form each of these portfolios. A and B both has a unique skewness level. Depending on the skewness combination of A and B, we constructed four categories:

i. Skewness of A and B are both greater than 1, denoted Pos & Pos
   The two-asset portfolios in this group are built by two stocks that both have skewness greater than 1.
ii. Skewness of A and B are both less than -1, denoted Neg & Neg
   The two-asset portfolios in this group are built by two stocks that both have skewness less than -1.
iii. Skewness of A and B are both 0, denoted Zero & Zero
   The two-asset portfolios in this group are built by two stocks that both have skewness equal or very close to 0.
iv. Skewness of A and B are larger than +1 and less than -1 respectively, denoted Pos & Neg
   The two-asset portfolios contain one stock that has skewness greater than +1 and the other less than -1.

Hence, we sort all the stock pairs we selected into two big categories (+0.6 to +0.8 and -0.6 to -0.8) and each category contains the four groups we stated above. For each portfolio in these groups, we calculate out the optimal point of the portfolio using minimum variance portfolio theory. “A portfolio of individually risky assets that, when taken together, result in the lowest possible risk level for the rate of expected return. Such a portfolio hedges each investment with an offsetting investment; the individual investor's choice on how much to offset investments depends on the level of risk and expected return he/she is willing to accept.”[19] The investments in a minimum variance portfolio are individually riskier than the portfolio as a whole. To minimize the risk is exactly what this paper is aiming to do so. We adopted this portfolio construction method in doing two-asset portfolio analysis.

For each group we have 10 portfolios with their minimum variance optimized. Then for each of these portfolios, we calculated the risk reduction level in terms of percentage. The risk reduction rate is quantified by comparing the optimized portfolio’s standard deviation to the smaller standard deviation of the two assets, which can be noted as the following:

Risk Reduction Rate= $[S_p – \text{Min}(S_1, S_2)]/ \text{Min}(S_1, S_2)$

In the above equation, $S_p$ is the minimum variance portfolio’s standard deviation and $S_1$, $S_2$ are the standard deviation of the two assets. The risk reduction rate can give us a sense of how much the standard deviation is reduced in each case when forming a portfolio and help us compare which skewness combination on average has the highest risk reduction rate. For correlation coefficient at 0.6 to 0.8, we have four groups and also for correlation coefficient at -0.6 to -0.8, we have the same four groups. Each group as we have picked has more than ten two-asset portfolios and we finally calculated the arithmetic mean of the risk reduction rate for each group. The result is as following:

**Table 1**

| Average Risk Reduction Rate | | | | |
|---|---|---|---|---|
| Groups | Neg & Neg | Zero & Zero | Pos & Pos | Neg & Pos |
| Correlation [-0.6, -0.8] | 28% | 43% | 47% | 41% |
| Correlation [0.6, 0.8] | 0.002% | 1.283% | 1.419% | 0.509% |

**Figure 1**

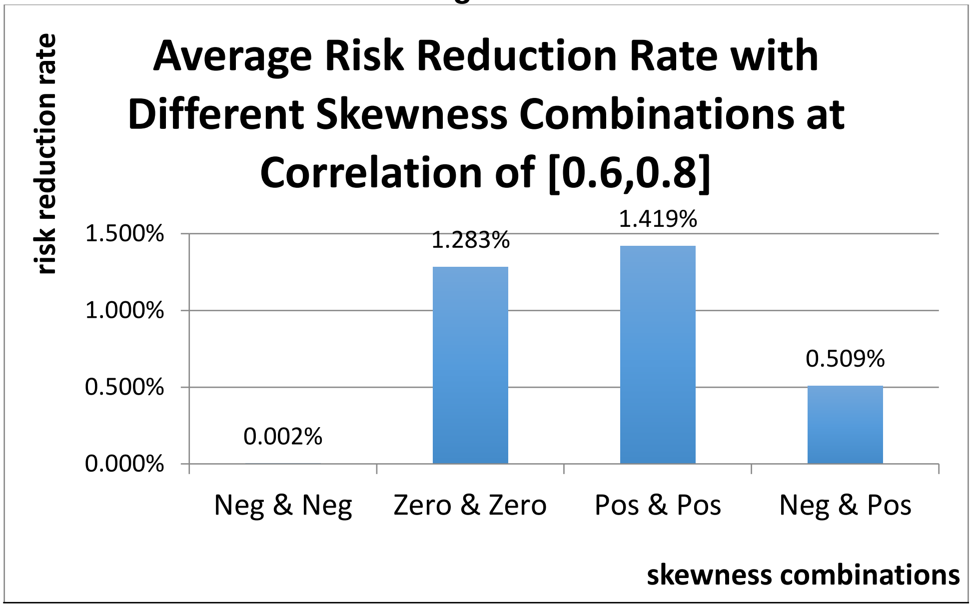
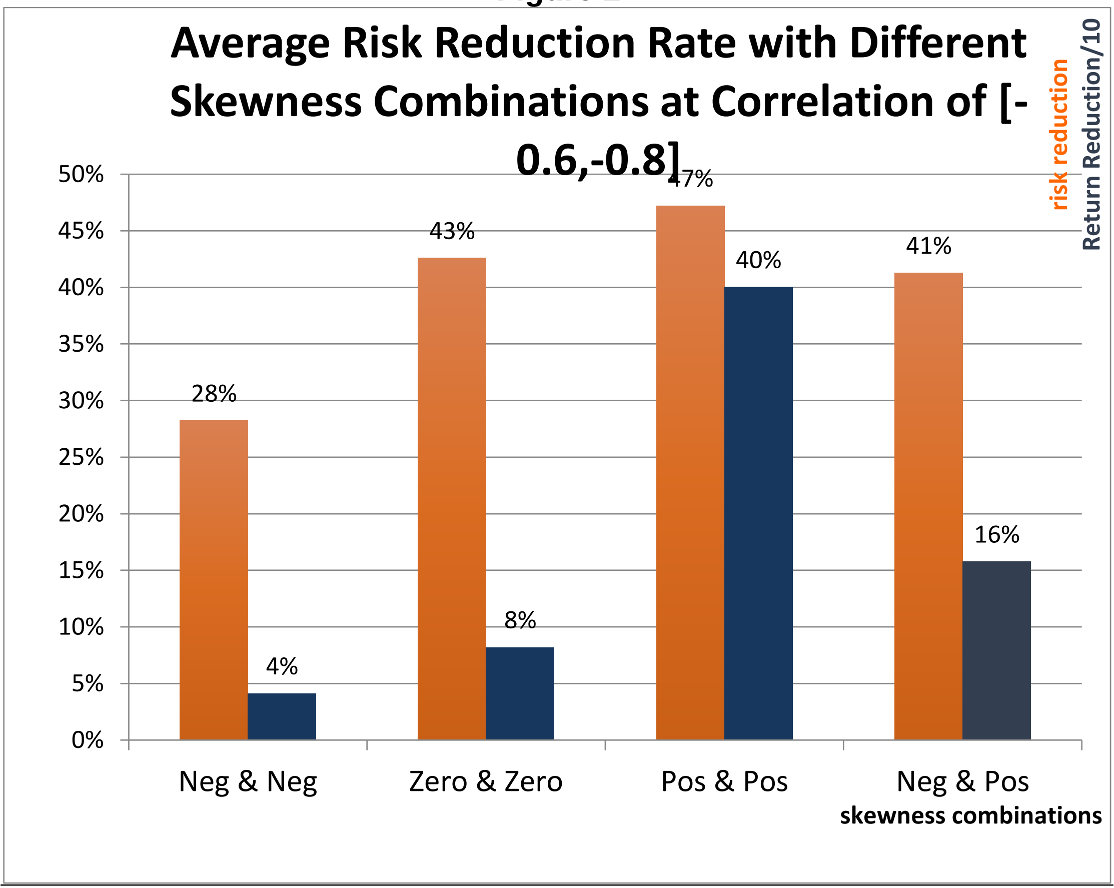



**Figure 2**




As we can see in the above graph, we also calculated the return reduction rate when the correlation coefficient is negative. When correlation coefficient is positive and especially at a high level of larger than 0.6, we know that the portfolio is very hard to diversify as the two assets in this kind of portfolio are highly correlated. Normally since the goal is to diversify risk, we do not consider involving too many highly correlated assets. Thus the study of the positive correlation coefficient is just to see if the result is consistent with the result from the negative correlation coefficient and see if there are any similar observations. This is the reason we focus more on negative correlation coefficient possibilities rather than positive.

The first and the most important observation is that no matter the correlation coefficient is either negative or positive, the combination of two negative skewed assets (group Neg & Neg) has the lowest risk reduction rate. When the two assets are positively correlated, there is almost no risk reduction for two negative skewed assets. This shows us the probability that when both assets are negatively skewed, the risk reduction is probably low. We find the same result in the category where correlation coefficient is [-0.6, -0.8]. From the graph above we can see that the average risk reduction rate for negatively correlated portfolios that consist two negatively skewed stocks is 28%, a percentage more than 30% lower than the second lowest reduction rate, which is at 41%. This supports our assumption that the combination with two negatively skewed assets has the lowest risk reduction rate.

On the level of two-asset portfolios, we confirm previous studies on skewness preference based on our empirical experiment and we find that negative skewness is not conducive in reducing the risk level of portfolio and thus negatively skewed assets are less preferable in the selection of two-asset portfolios. Two-asset portfolios that contain two negatively skewed assets have the smallest effect in reducing risk and the risk reduction rate for portfolios that consist one negatively skewed asset is also lower than those that do not have any negatively skewed asset.

## 4. The Findings

The goal of our study on two-asset portfolio is to help us better understand the behavior of multi-asset portfolios on skewness and risk reduction. As we have studied the simple version of portfolio construction regarding to skewness, we may generalize our findings from this simple version to more complicated cases where, using multiple stock selection methods such as the traditional method, etc., we construct portfolios with multiple stocks and study the relation between risk reduction and skewness on a higher level.

Therefore, from the finding we had on two-asset portfolios, (which indicated that negative skewness has a negative effect on portfolio risk reduction), we assume that negatively skewed assets are also having negative impact on the risk reduction of multi-asset portfolios. In other words, we assume that the more our portfolio is negatively skewed, the less there will be in portfolio risk reduction.  We construct regressions on the data gathered and try to find the relation between portfolio skewness and the portfolio risk reduction level. No matter if the result is consistent to our previous sample study or not, we will empirically present the outcome and try our best to discuss the theoretical explanation and implication behind the results.

As for data screening, we neglected industries that have less than 40 companies and it returns to us 25 different industries and a total of more than five thousand companies. From each of these industries, we randomly select one stock and as a result we will have a portfolio that contains twenty-five stocks. We then repeat this selection procedure and finally construct 40 portfolios using this method. For each portfolio, using their monthly returns on the year of 2011, we calculate the variables we need for our regression using equal weight (each company in the portfolio takes 1/25 of the portfolio). There are a total of six variables generated from the data. The variable names and descriptions are as described in table 2:

**Table 2**

| | Variable Abbreviation | Variable Name |
|---|---|---|
| Dependent Variable | riskreducrt | Risk Reduction Rate |
| Independent Variable | portskw | Portfolio Skewness |
| Control Variables | portreturn | Portfolio Return |
| | avrcorr | Average Correlation |
| | negskw | Number of negatively skewed asset in % |
| | posskw | Number of positively skewed asset in % |

We set our dependent variable to be the risk reduction rate and our independent variable to be the portfolio skewness. Next, we will introduce all the variables in a more detailed fashion and also illustrate the operations behind how they are generated. The dependent variable is the risk reduction rate of each portfolio. It represents on average, how much risk is reduced through the process of diversification. The reduction of risk is for the portfolio standard deviation to compare to the average standard deviation of all the individual assets' risk within the portfolio. The exact formula is as the following:

*Riskreducrt = (average standard deviation – portfolio standard deviation) / average standard deviation*

Here the average standard deviation is an estimate for the risk we will bear if we buy all the assets in the portfolio separately without any diversification. Usually, after a diversification using equal weight, the portfolio standard deviation should be less than or equal to the average standard deviation, which means that the worst case should be there is no risk diversification and if there is, the portfolio standard deviation should fall below the average standard deviation. Here the portfolio standard deviation is calculated by the weighted return of the portfolio. Using the 12 months return we get the portfolio standard deviation. In this sense, a higher risk reduction rate means that the portfolio did a better job in lowering the risk of the portfolio, which is more preferable in portfolio selection. In other words, the more risk diversification the higher risk reduction rate should be.

The independent variable, portfolio skewness, is the center of study in this paper and we are trying to discover the relation between skewness and our dependent variable, the risk reduction rate. In order to calculate the portfolio skewness, we first calculate the portfolio monthly return using equal weight. For each portfolio, we used equal weight to calculate the return of each month for the portfolio and then we have forty portfolios each has 12 entries reflecting their monthly data. Using this monthly data, we are able to calculate the skewness level of each portfolio.

There are a total of four control variables into testing. As we incorporate skewness as a third dimension into portfolio analysis, it becomes an MVS analysis, in which one must sacrifice one or two of the factors in order to achieve the level of the third. As we have risk reduction rate, which includes the portfolio risk in its calculation, skewness as another variable, we need to add portfolio return into the regression. This is the reason we take portfolio return, which is the mean return of the 12 months for each portfolio, into the regression and set it as a control variable.

In addition, the correlation coefficient is also a very important factor in portfolio selection. Before there is any attention on higher movements in stock returns, most investors look at the correlation coefficient and consider it to be the key factor in portfolio risk minimization. Correlation coefficient describes the level of co-movement between two sets of data and reflects how strong the two move together in terms of increase and decrease of returns. Thus we introduce average correlation coefficient as another control variable.

Since correlation coefficient only exists when considering two individual assets, it is hard to find the correlation coefficient of a portfolio that has over twenty individual stocks. Therefore, in order to represent the average correlation level among all the assets within a certain portfolio, we introduce the variable "average correlation" into the analysis. To calculate this variable, we must first have the monthly returns of all the individual stocks in this portfolio and construct a correlation matrix[20]. Each cell in the correlation matrix shows the correlation coefficient between the company in that column and the company on that certain row. The diagonal of this matrix should all be one as one always represents the correlation coefficient between a certain asset and itself. In this way, we are able to find the correlation between each of the two assets in this portfolio. Then, we take the mean value of all the correlation coefficients in this portfolio and that gives us the average correlation coefficient of this certain portfolio. In this way, we adopt the calculation process to all 40 portfolios and it gives us 40 entries of data for this variable as well.

The last two control variables are respectively the proportion of the positively and negatively skewed assets in the portfolio. As each portfolio has 25 assets, if for example in one portfolio we have 10 companies that are positively skewed, then the number for this variable should be 10/25 = 0.4 (40%).

**Table 3**

| Dependent Variable | Riskreducrt | t | P-value |
|---|---|---|---|
| portskw | -0.0315 | -2.6* | 0.014 |
| portreturn | 0.057 | 0.08 | 0.937 |
| avrcorr | -0.563 | -6.29* | 0.000 |
| negskw | 0.057 | 0.51 | 0.61 |
| poskw | 0.044 | 0.5 | 0.62 |
| Cons | 0.575 | 6.85* | 0.000 |
| $R^2$ | 0.657 | | |

Using the data we gather and import them into STATA, we first run an OLS regression and form table 3.5. The adjusted R-squared value is at 0.6059. Note that the p-value of average correlation coefficient is at 0. This means that the effect of average correlation coefficient in risk reduction is also very significant. The p-value for constant is at zero as well. This indicates that our dependent variable is fully explained by the variables so far and that we are not missing any control variable. Even though the result is significant at this point, we still need to continue and go through all the necessary tests in order to eliminate any potential problem and then reach a final regression result.

Now since we have data of all different portfolios, we are actually dealing with cross section data. Cross-sectional data refers to data collected by observing many subjects (such as individuals, firms or countries/ regions) at the same point of time, or without regard to differences in time. Analysis of cross-sectional data usually consists of comparing the differences among the subjects. Our data set fits this profile as we are comparing the differences among the 40 portfolios.

There are a few notable observations. First of all, the output data indicates that the portfolio skewness and portfolio average correlation are the two key factors affecting the portfolio risk reduction. For the correlation, not only that the p-value indicates the significance of this variable, but also that the coefficient of this factor is -0.5, which is the largest coefficient comparing to the other ones in the test. The negative sign indicates that there is a negative relation between portfolio average correlation and risk reduction. In other words, this finding strongly supports the Markowitz's theory that an increase in correlation will result in a decrease in portfolio reduction rate. Intuitively speaking, the more we choose assets that are highly correlated to each other into our portfolio, the less risk reduction we will have.

For the relation between portfolio skewness and the risk reduction, the research result is fairly interesting. We can see that the p-value for this independent variable is also close to zero. This tells us that in our data set, the level of portfolio skewness does affect the portfolio risk level and hence as Mr. Markowitz and other scholars predicted, higher movements should indeed be taken into account. However, the coefficient of the independent variable is actually negative. This result is the opposite of our assumption that we made from the findings with two-asset portfolios. We assumed that the relation between the portfolio skewness and portfolio risk reduction rate should be positive, meaning that the more positive the portfolio's skewness is, the higher rate of risk reduction one should have. In the multi-asset setup, the more we choose negatively skewed assets for our portfolio, a better risk reduction yield we will have.

A very important fact that we need to keep in mind is that this selection method only makes our portfolio more optimized. This means that because we pick stocks only from industries that have relatively lower correlation coefficients, the portfolios we have, on average, will be better off in terms of increasing risk reduction rate compared to the previous portfolio selection method. But the study on the general relation between portfolio skewness and risk reduction rate is constrained and might be biased. Thus no matter what the result is through the study on this specific selection method, one should keep a speculative attitude and only try to see the possible implications through the results.

The data has covered the firm from 25 industries. Using averaged monthly return, we construct a correlation coefficient matrix that shows the correlation between each of the two industries. Then, for each industry, we calculate the mean of the correlation coefficients this industry has to the rest 24 industries. We use this calculation to estimate the average correlation of each industry to the rest of the 25 industries. The correlations between each industry are all quite high but still there are several industries that are relatively lower than the rest. Using this ranking and historical data from Morningstar, we selected 5 industries that are relatively less correlated and have relatively better rate of return in the last three years. On average, they all have industry correlation coefficient less than 0.9 between each other. There may be many other combinations for us to pick five industries in this way but here we will use the one we find. The industries we find are as following:

**Table 4**

| SIC | Industry Name |
|---|---|
| 20 | FOOD & KINDRED PRODUCTS |
| 36 | ELECTRONIC & OTHER ELECTRICAL EQUIP |
| 51 | WHOLESALE TRADE-NONDURABLE GOODS |
| 63 | INSURANCE CARRIERS |
| 80 | HEALTH SERVICES |

By picking from only these five relatively uncorrelated industries, we can lower the correlation coefficient between the stocks we pick in general and help us diversify more risk, which is different from the previous portfolio selection method that we may choose stocks though from different industries but are still highly correlated. Now from each of these five industries, we randomly select five stocks. In this way, we will again form portfolios with 25 stocks. Repeating the same procedure, we again constructed forty portfolios and each contains the 5 industries we fixed and 25 stocks.

We observe that there is no obvious outlier and again the general trend of the relation between portfolio skewness and risk reduction rate is negative. We run the OLS regression again by those five industries with low correlations and the result is as following:

**Table 5**

| Dependent Variable | Riskreducrt | t | P-value |
|---|---|---|---|
| portskw | -0.009 | -0.96 | 0.342 |
| portreturn | 0.726 | 1.34 | 0.191 |
| avrcorr | **-0.877** | **-13.13*** | 0.000 |
| negskw | -0.073 | -0.91 | 0.368 |
| poskw | **-0.146** | **-2.09*** | 0.045 |
| Cons | 0.825 | 12.48* | 0.000 |
| $R^2$ | 0.8827 | | |

This time, we find that the p-value of the independent variable is larger than 0.05. However, our Adjusted R-squared value is at a high of 0.8654. Still, we yet cannot draw any conclusion so far since there may involve other problems in the regression that we need to test. We now move one to test for multicolinearity in the data. We adopted the test of variance inflation factor for multicollinearity (VIF) to check and there is no multicolinearity in the model.

It is reasonable that the portfolio skewness is insignificant in this specific selection method. Comparing to the previous portfolio selection method, selection by industry narrowed our vision to only five industries. This probably will make the regression biased, as our selected sample is limited. However, since the industry selection method is widely used in portfolio selection, the result under this method is still valuable as one can study the behavior of skewness to the portfolio performance under this specific portfolio selection method.

Using this alternative portfolio selection method we see that the result is not entirely consistent with the previous regression outcome. One commonality is that both results show that indisputably the average correlation coefficient of the portfolio has a strong negative effect to the portfolio risk reduction rate. A new discovery from this selection method we see is that the variable that indicates how much percentage of positively skewed assets are selected in the portfolio is significant. It has a negative relation to our dependent variable since the coefficient is at -0.14. This means that the more we select positively skewed stocks to be in our portfolio, the lower risk reduction rate we will have. This result though different from the previous section, also implied that a positive skewness is less preferred.

## 5. Conclusions

Even though as many scholars have agreed that positive skewness is always preferred, the empirical testing result generated in this paper proves that the stock selection criteria needs more consideration. The result shows us counter examples in which negative skewness is actually preferred in risk reduction. Both selection methods indicate that an increase in either portfolio skewness or number of positively skewed assets in the portfolio will actually decrease the risk reduction rate and hence give us a less optimized portfolio. These findings rejected the previous study results that positive skewness is always preferred for investor who seeks to lower their portfolio risk. We should certainly not generalize such assertion since we only focused on one specific year of the stock market and we only adopted two stock selection methods. However, this study still has its significant implication that when using portfolio optimization it is not always the case that positive skewness should be desired. Sometimes, negative skewness may result in a better improvement in portfolio risk reduction. This discovery rings a bell to the current investors that when including skewness in portfolio optimization, one should at first run tests on the stock market and see its behavior in at least the previous year in order to have a better understanding of whether positive or negative skewness is preferred in the portfolio risk reduction at the moment. After a short study, the investor can optimize his portfolio using his model and findings on the skewness effects. Therefore, the paper concludes that it is not always true that positive skewness is preferred in risk minimization. Sometimes, as our experiment result tells us, more negative skewness can actually decrease the portfolio risk. Investors' discretions are highly advised when trying to incorporate skewness into portfolio optimization, as the effect of skewness is not always positive.

The paper agrees that there are potential problems that we need to carry out further study to improve the research results. Because there is massive calculation required to process the data and generate variables we construct, one problem the research faces is that the spectrum of data in this study is limited. In limited time of research, we only manage to study the skewness behavior in one year. This hinges us to generalize us to generalize our research result pertaining to the other years. We strongly believe that more computer programming should be involved and it will effectively reduce our data processing time and effort. Scholars have argued that skewness does not tend to last over time. Therefore, if we wish to study skewness not in a cross-sectional form of data but as panel data that spans the past decade, we also need to closely study the behaviors of skewness over time. The persistence of skewness could affect our findings.

Secondly, the paper still holds a speculative attitude towards the research result. If we look at the equation of skewness, and the equation of our risk reduction rate is $RR = \frac{\bar{\sigma}-\sigma}{\bar{\sigma}} = 1 - \frac{\sigma}{\bar{\sigma}}$ where $\mu$ is mean return, $\sigma$ is portfolio standard deviation and $\bar{\sigma}$ is average risk. We can see that if we increase the skewness, holding the returns constant, the standard deviation will decrease and hence our risk reduction rate should increase. Hence theoretically, it is reasonable to assert that skewness should have a positive relation to portfolio risk reduction. However, our paper shows the opposite and this may imply that we are missing certain control variables that describes skewness behaviors. A look at the proportion of positively and negatively skewed assets in the portfolio composition is a good start. More control variables such as market fluctuation indicators and dummy variables for different industries could be included and one can see if it describes the behavior better.

There is also another possible explanation to the result we have. Skewness can describe the shift of center of mass of a distribution but it is possible that for two distributions that have the same skewness level, the thickness of their tail is different. Both distributions are negatively skewed but the one on the left has a fatter tail skewed to the left. This compares to the one the right has more extremely negative returns and is less preferred. In our paper, the distribution is approximately the one on the right, which has very few negative results and a lot of small positive returns. This makes negative skewness hence more desirable. To look at this possibility, we need to find variables that can depict the thickness of the tail and help up control for the results, which is also a very plausible approach for us in the future.

## Endnotes

[1] Other contributors to this argument: Kraus and Litzenberger (1976), Harvey and Siddique (2000)
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[18] See complete descriptive in the appendix
[19] Jason Hsu (2011) " Quantitative Asset Management Notes on Minimum Variance Portfolios"

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