International Review of Business Research Papers
Vol. 12. No. 1. March 2016 Issue. Pp. 97 – 109

Power-Log Portfolio Optimization for Managing Downside Risk

Jivendra K. Kale* and Arnav Sheth**

This research paper tests the effectiveness of Power-Log optimization for managing the downside risk of investment portfolios. It uses Power-Log utility functions, which are based on tenets of behavioral finance, to give investors the ability to build downside protection directly into a portfolio. Comparing optimal Power-Log portfolios with matched mean-variance efficient portfolios, we find that the optimal Power-Log portfolios have lower downside risk, while delivering higher geometric average return. They also provide much better downside protection against unanticipated market shocks, such as the one in 2008, in contrast to the disastrous performance for matched mean-variance efficient portfolios. Power-Log optimization succeeds in managing downside risk effectively, while mean variance analysis fails to protect investors from such risk.

JEL Code: G11

1. Introduction

In this study we test the effectiveness of using Power-Log utility functions for constructing investment portfolios with low downside risk, and compare it to the widely used mean-variance analysis method of portfolio construction. Mean-variance analysis has been the standard method for portfolio selection since Markowitz (1953) introduced the idea of a mean-variance efficient portfolio. It works well when asset returns are approximately normal, because investor preferences can be completely specified by mean and variance, regardless of the investor's utility function, but It performs poorly when asset returns are skewed and have fat tails, as they are for options and some other assets. Pedersen (2001) points out that mean variance analysis has a further shortcoming in that it ignores the positive skewness in portfolio returns that investors find desirable, and treats high upper tail returns as undesirable since they produce higher variance of return.

Using market data, we show that portfolio optimization with Power-Log utility functions is very effective in controlling downside risk as well as generating better portfolio growth than the widely used mean-variance analysis. Power-Log optimization is also different from other methods of portfolio construction since it uses a single parameter, the downside power, to model the collection of investor preferences for expected return, variance, skewness and higher moments. This makes Power-Log optimization a simple, elegant and powerful method for constructing portfolios that investors desire.

The following sections describe the significant literature in the field, our model, our findings, and conclude with a summary.

*Dr. Jivendra K. Kale, Finance Department, Saint Mary's College of California, Moraga, USA
Email : jkale@stmarys-ca.edu
**Dr. Arnav Sheth, Finance Department, Saint Mary's College of California, Moraga, USA
Email: aas3@stmarys-ca.edu

2. Literature Review

Existing methods of portfolio construction that focus on downside risk, either trade off mean return for a portfolio against some measure of downside risk, such as Value at Risk (RiskMetrics 1996) or lower partial moments such as semivariance (Markowitz 1959, Jarrow and Zhao 2006) and Conditional Value at Risk (Rockafellar and Uryasev 2000), or use multiple objectives (Anagnostopoulos and Mamanis 2010), or focus on below target returns (Fishburn 1977, Bawa 1978), or use dynamic hedging to insure portfolios (Rubinstein and Leland 1981, Leland and Rubinstein 1988), or use fuzzy set methodology (Vercher, Bermudez and Segura 2007). All these ad-hoc methods are based on arbitrary tradeoffs between risk and return, and are not based on investor preferences incorporated in utility functions, and therefore produce results that are arbitrary. Studies using prospect theory as the basis of normative portfolio construction have found that the negative skewness in returns of the resulting optimal portfolios, is counter to the well known investor preference for positive skewness in portfolio returns (Cremers, Kritzman and Page 2005). In contrast, Power-Log optimization uses investor preferences expressed in Power-Log utility functions that are based on tenets of behavioral finance and have a strong grounding in economic theory (Barberis and Thaler 2003, Kale 2006), which corrects the arbitrariness of the preceding research.

3. The Methodology and Model

The following description of the methodology and model is based on Kale (2006). Tversky and Kahneman (1991) represent prospect theory's investor preferences with an S-shaped utility function, with the following characteristics:

> "Outcomes of risky prospects are evaluated by a value function that has three essential characteristics. Reference dependence: the carriers of value are gains and losses defined relative to a reference point. Loss aversion: the function is steeper in the negative than in the positive domain; losses loom larger than corresponding gains. Diminishing sensitivity: the marginal value of both gains and losses decreases with their size. These properties give rise to an asymmetric S-shaped value function, concave above the reference point and convex below it."

Tversky and Kahneman's S-shaped utility function's decreasing sensitivity to losses might be appropriate as a descriptive model for explaining speculative behavior, but it is not appropriate as a normative model of investor preferences for constructing portfolios. The experiments in support of prospect theory, such as those described in Kahneman and Tversky (1979), are designed as gambles where only a small fraction of an individual's wealth is at stake and do not apply to investors who are investing significant portions of their wealth, for example for their retirement funds. Nick Leeson's desperate actions, where he took bigger and bigger risks as his losses increased and eventually bankrupted Barings Bank in 1995, might be explained as speculative behavior that is consistent with diminishing sensitivity to losses, but his case presents an agency problem since he was speculating with the bank's money and not his own. Investors typically invest for their future well-being, and are unlikely to take positions where there is the possibility of losing their entire wealth. Cremers, Kritzman and Page (2005) also found that, "… investors with S-shaped preferences are attracted to kurtosis as well as negative skewness," which is contrary to the well-known investor preference for positive skewness in returns.

Multiperiod portfolio theory offers an avenue to operationalize most of the tenets of prospect theory in a normative model. When the goal is to maximize portfolio growth over time, multiperiod utility theory leads us to the log utility function (Kelly, 1956), and it is defined as,

$$U = \ln(1+r) \qquad (1)$$

Where,

r	portfolio return
ln	natural log function

Like the log utility function, power utility functions also have desirable multiperiod characteristics. A power utility function is defined as,

$$U = \frac{1}{\gamma}(1+r)^{\gamma} \qquad (2)$$

Where,

r	portfolio return
γ	power, is less than or equal to 1

The log utility function is a special case of the power utility function with power gamma set to zero (Grauer and Hakanssson 1982). A power utility function with power less than zero represents greater risk aversion than does the log utility function. The more negative the power, the greater the penalty for losses, which implies greater loss aversion; unfortunately, it also implies that a smaller value is associated with gains. Power-Log utility functions combine the maximum growth characteristics of the log utility function on the upside, with the scalable loss aversion property of power utility functions on the downside. Kale (2006) defines a Power-Log utility function as,

$$\begin{aligned} U &= \ln(1+r) \quad \text{for } r \geq 0 \\ &= \frac{1}{\gamma}(1+r)^{\gamma} \quad \text{for } r < 0 \end{aligned} \qquad (3)$$

Where,

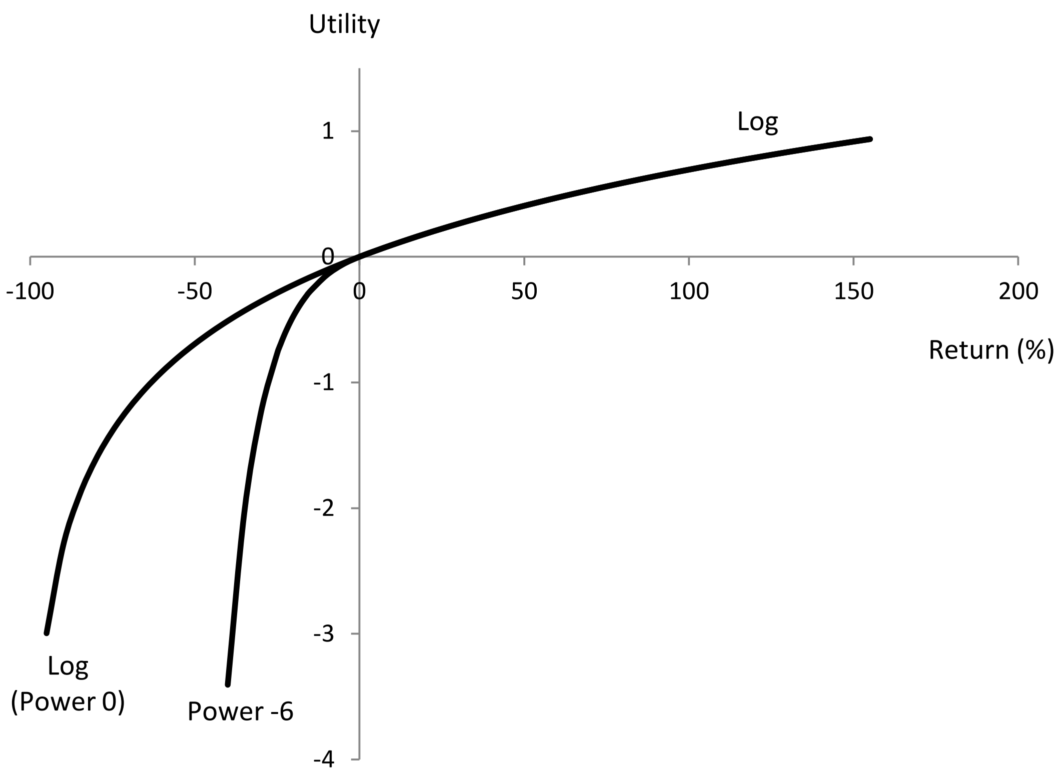
r	portfolio return
γ	power, is less than or equal to 0

Figure 1 shows an example of Power-Log utility functions with downside powers zero and –6. The figure reflects a positive linear transformation of the power function of Equation 3, which does not change the utility associated with the function. Selecting a downside power of zero is equivalent to using a log utility function for losses, which will result in the construction of a maximum growth portfolio since the utility function for gains is also a log utility function. Investors can increase the level of downside protection they build into their portfolios by reducing the downside power. Lower, more negative, values of the downside power represent greater aversion to losses since the penalty for losses increases, while the value associated with gains is unchanged. The Power-Log utility function with downside power -6 represents greater loss aversion than the one with downside power zero, but both have the log utility function for gains.

Power-log utility functions are characterized by an increasing sensitivity to losses as the size of losses increases, which represents risk averse behavior that is modeled with a concave function. As a result Power-Log utility functions are concave for the entire domain and model risk averse investor behavior across the board. When compared to Kahneman and Tversky's

S-shaped utility function, the Power-log utility functions' increasing sensitivity to losses is a better representation of investor preferences for constructing portfolios. In fact, for Power-log utility functions the utility associated with a 100% loss is negative infinity. The Power-log utility functions will not allow the selection of a portfolio where a 100% loss has a probability greater than zero, and this is not true of prospect theory's S-shaped utility function, which is risk seeking where losses are concerned. Power-Log utility functions conform to the Friedman-Savage (1948) axioms for a risk averse utility function, and work very well as a normative model for representing investor preferences.

Figure 1: Power-Log Utility Function with Downside Power -2 and -20



To construct optimal portfolios, the expected utility criterion developed by Von Neumann and Morgenstern (1944), and Savage (1964) gives us the following one-period optimization problem for selecting assets weights, w_i:

Maximize

$$E(U) = \sum_s p_s U_s \qquad (4)$$

Where,

s scenario s, and the summation is over all scenarios
p_s probability of scenario s
U_s utility of portfolio return r_s in scenario s, where utility is defined by the Power-Log function in Equation 3

The portfolio return, r_s, is calculated as an investment weighted average of the returns to the assets in the portfolio,

$$r_s = \sum_i w_i r_{is} \tag{5}$$

Where,

i asset i, and the summation is over all assets in the portfolio
w_i investment weight of asset i in the portfolio
r_{is} return to asset i in scenario s

We construct optimal portfolios using Power-Log utility functions and rebalance them once a year, following the methodology of Kale and Sheth (2012). The data is for years 1996-2009, which include wide swings in the stock market, and the portfolio optimization algorithms are from Financiometrics Inc. In each year the portfolio is purchased on the December expiration date of the call option and sold on the December expiration date of the following year, which is the rebalancing date. In other words, portfolio rebalancing is synchronized with the December expiration dates of the call options. For example, in the first year, an optimal portfolio is constructed and purchased at the beginning of the year on December 20, 1996, the expiration date in December 1996. It is held for about a year and sold on December 19, 1997, the expiration date in December 1997, when a new optimal portfolio is constructed and purchased for the following year. To construct the optimal portfolio for the first year on December 20, 1996, we select the closest to the money call option that expires at the end of the year on December 19, 1997, for inclusion in the portfolio. We calculate each call option's purchase price as the average of the closing bid and ask on the purchase date at the beginning of the year. The S&P500 index data is from yahoo.com, and the options data is from CSIData.

To construct an optimal portfolio with a Power-Log utility function, we need the joint distribution of returns for all assets that can be included in the portfolio. We tested the annual return distribution of the S&P500 index for lognormality using the Lilliefors, Jarque-Bera and χ^2 tests, and found that we could not reject the hypothesis of lognormality at the 5% level of significance. Given that annual S&P 500 returns are approximately lognormal, we simulate the distribution for the log return as a normal distribution for each year. The Black-Scholes implied volatility calculated from the closest to the money call option on the S&P500 index gives us a market forecast for the standard deviation of return for the following year. Since there is no universally accepted forecast for the mean of the distribution, we use 10% as the mean log return for the index, which is its approximate value for the postwar period.

To avoid the problem of fickle randomly generated points, we use deterministic simulation instead of Monte Carlo simulation (Kale 2011). We generate one million observations by dividing the domain of the lognormal return distribution into one million intervals of equal probability, and then calculate the median for each interval. Using one million observations gives us sufficient and consistent representation in the tails of the return distribution.

To generate the call option returns that correspond to the simulated S&P500 index returns for a given year, we use the market value for the index at the beginning of each year, and the simulated index returns to generate index values at the end of the year. Next we calculate expiration values for the call option based on these index values and the strike price. We use these end of year expiration values and the beginning of year purchase price of the call to calculate call returns that correspond to the S&P500 index returns. The extreme positive skewness of the option return distribution gives us the ability to incorporate options in portfolios to control the damage of black swan events on the downside, and take advantage of black swan events on the upside.

We combine the riskless return, which is based on the one-year constant maturity treasury yield, with the simulated return distributions for the S&P500 index and the call option, to create an ex-ante joint return distribution for the three assets at the beginning of each year. This ex-ante joint return distribution changes from one year to the next, as the treasury yield and the implied volatility of the S&P500 index change.

We use the simulated ex-ante joint return distribution at the beginning of each year to construct the optimal portfolio for that year, by solving the optimization problem defined by equations 3, 4 and 5 directly. We do not convert it to an equivalent lower partial moment problem. The direct solution of the optimization problem avoids the necessity of making any additional assumptions about the nature of the portfolio's probability distribution of returns, or investor preferences. To provide a context for evaluating the performance of the optimal Power-Log portfolios, we construct mean-variance efficient portfolios with ex-ante expected returns matched to those of the optimal Power-Log portfolios, since mean-variance analysis is the most widely used and familiar method of portfolio construction. In the sections that follow we compare the portfolio compositions and risk and return characteristics of portfolios constructed with the two techniques.

4. The Findings

We start by constructing optimal portfolios with the log utility function, by setting the downside power to zero in the Power-Log utility function. In theory, the log utility function maximizes portfolio growth, and the resulting portfolios are the riskiest in our sets of portfolios. Table 1 shows the Black-Scholes implied volatility at the beginning of each year, and the compositions of the optimal portfolios constructed at the beginning of each year. For example, the 12/20/1996 purchase date is the date on which the optimal portfolios are constructed for the first year. To make it easier to interpret the results in this section, we impose a "no short sales" constraint on the S&P500 index and the call option. The ex-ante expected return shown in the last column is calculated from the optimal asset weights and the simulated ex-ante joint return distribution at the beginning of each year.

Every portfolio in Table 1 is very risky. The December 1996 portfolio has an investment of 32.14% in the call option, which potentially could finish out of the money and result in a total loss for that position. It is of course, counterbalanced by a 67.86% investment in the riskless asset, which will ensure the portfolio's survival. In every year the optimal portfolio has a large weighting in the call option, with a minimum of 17.85% in December 1999, and a maximum of 46.72% in December 2004. The four largest positions in the call option are over 40% in the years 2003-2006, when the Black-Scholes implied volatility was the lowest, ranging from 14.14% to 15.41%. When the implied volatility was high, which is usually true when the market consensus is that there is high lower-tail risk for S&P500 returns, the call option position was smaller, resulting in less risky portfolios. The geometric average ex-ante expected return over the 1996-2009 period is 33.25%, which reflects the leverage provided by the option positions.

Table 1: Optimal Power-Log Portfolios (Downside Power 0)

Year	Purchase Date	B-S Implied Volatility (%)	Riskless Weight (%)	S&P500 Weight (%)	Call Option Weight (%)	Expected Return (%)
1	12/20/1996	18.12	67.86	0.00	32.14	27.97
2	12/19/1997	23.00	74.41	0.00	25.59	20.94
3	12/18/1998	25.85	36.04	45.94	18.02	20.82
4	12/17/1999	23.94	49.36	32.80	17.85	17.81
5	12/15/2000	24.73	55.32	24.11	20.57	18.95
6	12/21/2001	21.79	6.34	68.39	25.27	32.86
7	12/20/2002	25.59	17.22	58.44	24.34	32.48
8	12/19/2003	15.41	-14.93	73.60	41.33	68.40
9	12/17/2004	14.57	53.29	0.00	46.72	58.57
10	12/16/2005	14.61	57.09	0.00	42.91	43.38
11	12/15/2006	14.14	57.31	0.00	42.69	41.27
12	12/21/2007	22.99	64.41	3.94	31.65	29.46
13	12/19/2008	39.67	43.39	35.58	21.03	29.32
Geometric Average						33.25

Next we construct matched mean-variance efficient portfolios, such that their ex-ante expected returns equal the ex-ante expected returns for the optimal Power-Log portfolios at the beginning of each year. Table 2 shows the compositions of the matched mean-variance efficient portfolios. Coincidentally, the asset weights for the mean variance efficient portfolios are identical to those for the optimal Power-Log portfolios in December 1996 and 2006. For all other years the mean-variance efficient portfolios are very different from the optimal Power-Log portfolios, and most of them have zero exposure to the call option, since the positive skewness of the call option returns is not valued, and upper tail returns increase risk in a mean-variance optimization. Like the optimal Power-Log portfolios in Table 1, the mean-variance efficient portfolios in Table 2 are very risky, with some extremely large leveraged positions in the S&P500 index. For example, the December 2003 portfolio has zero exposure to the call option, but has a 636.05% weighting in the stock index and a 536.05% short position in the riskless asset, to provide the necessary leverage to achieve an ex-ante expected return of 68.40% for that year.

Table 2: Mean-Variance Efficient Portfolios Matched to Optimal Power-Log Portfolios (Downside Power 0)

Year	Purchase Date	Ex-Ante Expected Return (%)	Riskless Weight (%)	S&P500 Weight (%)	Call Option Weight (%)
1	12/20/1996	27.97	67.86	0.00	32.14
2	12/19/1997	20.94	-19.75	108.47	11.29
3	12/18/1998	20.82	-67.37	167.37	0.00
4	12/17/1999	17.81	-52.84	152.84	0.00
5	12/15/2000	18.95	-53.26	152.54	0.72
6	12/21/2001	32.86	-180.66	280.66	0.00
7	12/20/2002	32.48	-143.32	243.32	0.00
8	12/19/2003	68.40	-536.05	636.05	0.00
9	12/17/2004	58.57	-521.05	621.05	0.00
10	12/16/2005	43.38	33.37	25.81	40.82
11	12/15/2006	41.27	57.31	0.00	42.69
12	12/21/2007	29.46	-134.06	230.67	3.39
13	12/19/2008	29.32	-51.25	151.25	0.00
Geometric Average		33.25			

While some investors, most notably Paul Samuelson (1971), might be willing to accept the risk associated with portfolios constructed with the log utility function, which is the Power-Log utility function with downside power zero, the risk is unacceptable for the vast majority of investors. As shown in Table 1 the smallest investment in the call option was 17.85% for the optimal Power-Log portfolio in year 4, and the largest investment was 46.72% in year 9. These are very large investments in a derivative, and carry a lot of risk. Portfolio risk can be reduced by reducing the downside power, by making it more negative, which increases the penalty for losses. We construct optimal Power-Log portfolios for thirty-four downside powers ranging from 0 to -50, and also construct mean variance efficient portfolios to match the ex-ante expected returns of the optimal Power-Log portfolios.

To evaluate the performance of the optimal portfolios, we calculate their geometric average return for the 1996-2009 period. The portfolio return in each year is calculated from the realized returns for the riskless asset, the stock index and the call option on the index. The riskless return is based on the one-year constant maturity treasury yield at the beginning of each year. The stock index return is the total return for the S&P500 index. During 1996-2009 it ranged from -38.32% to 28.43%. To calculate the call option return we use the purchase date price of the option, and the expiration value of the option at the end of the year, because the expiration value gives us a more reliable valuation of the call than the market price on the expiration date of the option. The call option finished out of the money in four of the thirteen years with a return of -100.00%, and had a best return of 202.74%, which reflects the positive skewness in the call option returns.

To measure downside risk we use the negative semideviation of returns below zero. It is a better measure of risk than standard deviation when returns are skewed, since it focuses on downside returns only (Markowitz, 1959). It is defined as,

$$NegativeSemideviation = \sqrt{\sum_s p_s \left(r_s^-\right)^2}$$

Where,

r_s^- portfolio return in scenario s if it is negative, zero otherwise, and the sum is over all scenarios.

p_s probability of scenario s.

Table 3 shows the geometric average return and negative semideviation of return for optimal Power-Log portfolios constructed with five downside powers from 0 to -50, and also for the matched mean-variance efficient portfolios. For downside power zero, which gives us the riskiest portfolios, the geometric average return is 6.16% for optimal Power-Log portfolios, versus -12.26% for the matched mean-variance efficient portfolios. This dramatic difference in favor of the optimal Power-Log portfolios is a result of the large losses that were avoided by the optimal Power-Log portfolios, but were devastating for the mean-variance efficient portfolios. This is also reflected in the negative semideviation of returns, which is 16.89% for the optimal Power-Log portfolios, while it is more than double that at 33.99%, for the matched mean-variance efficient portfolios. The much smaller losses for optimal Power-Log portfolios during market downturns also result in a lower standard deviation of returns and greater positive skewness in returns, than for the matched mean-variance efficient portfolios.

Table 3: Geometric Average and Negative Semideviation of Realized Returns

		Optimal Power-Log		M-V Efficient	
	Downside Power	Geo. Avg.	Negative Semideviation	Geo. Avg.	Negative Semideviation
1	0.00	6.16	16.89	-12.26	33.99
2	-0.60	6.38	13.27	-0.37	27.95
3	-3.00	5.90	7.63	4.64	17.43
4	-9.00	5.19	3.84	5.12	9.99
5	-50.00	4.35	0.95	4.37	3.94

Figure 2 shows the geometric average return and negative semideviation of return for optimal Power-Log portfolios constructed with thirty-four downside powers ranging from -50 to 0. The geometric average return is consistently positive for all portfolios ranging from the most conservative to the riskiest. It rises steadily as downside power increases toward zero, which is remarkable for downside powers greater than -10, since the negative semideviation increases substantially for those downside powers. The large losses associated with big market downturns are contained, and do not damage overall performance. As the downside power gets really close to zero, the geometric average return dips a little, which suggests that a little additional downside protection can be better for portfolio growth than using the log utility function (downside power zero).

Figure 2: Optimal Power-Log Portfolios

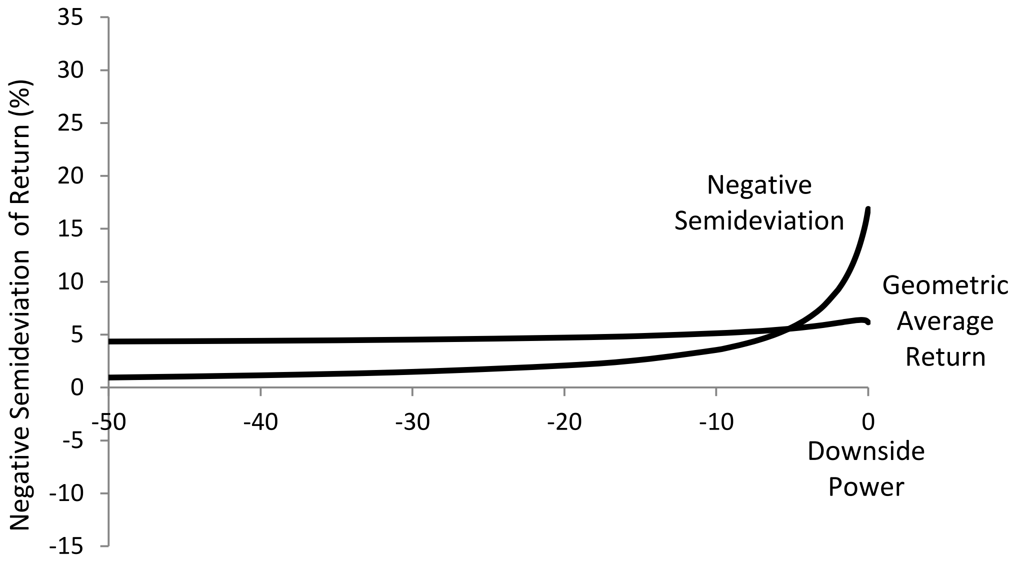
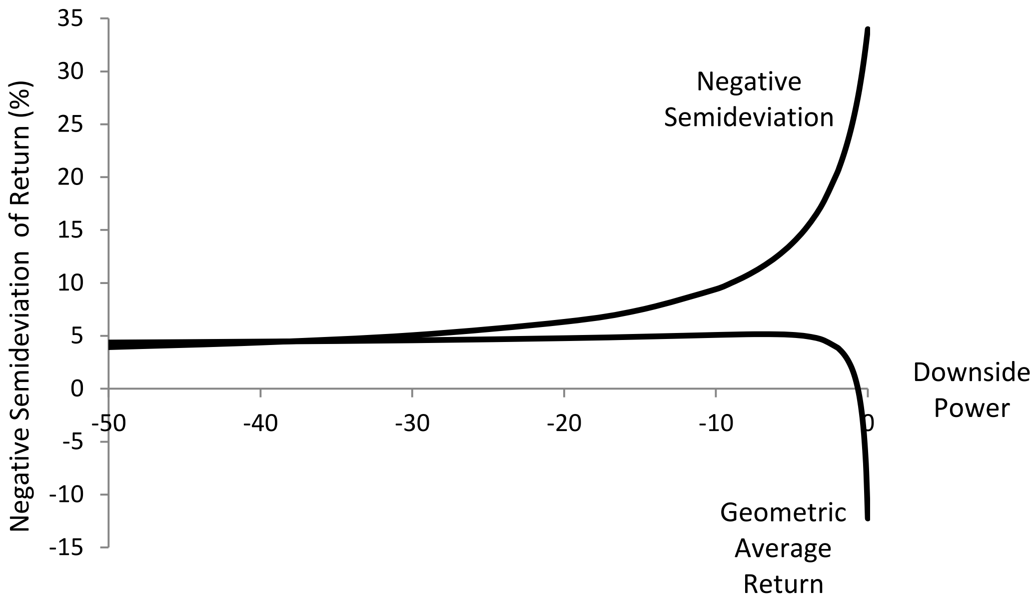


Figure 3 shows the geometric average return and negative semideviation of return for the mean-variance efficient portfolios matched to the optimal Power-Log portfolios. Unlike the optimal Power-Log portfolios whose geometric average return keeps increasing with downside power until the downside power reaches -0.5 as shown in Figure 2, the geometric average return for the matched mean-variance efficient portfolios starts dropping when downside power reaches -7.0, becomes negative for a downside power of approximately -0.65, and falls precipitously to -12.26% for downside power zero. The corresponding negative semideviation increases dramatically in comparison to the optimal Power-Log portfolios and goes all the way up to 33.99% for downside power zero. Unlike the optimal Power-Log portfolios, the large losses associated with big market downturns are not contained in the riskiest mean-variance efficient portfolios, and damage their overall performance substantially.

Figures 2 and 3 show that the performance of optimal Power-Log portfolios is consistently superior to the performance of matched mean-variance efficient portfolios across the risk spectrum. In particular, for the riskiest portfolios the performance of the optimal Power-Log portfolios was stellar compared to the disastrous performance of the matched mean-variance efficient portfolios. These findings validate the Power-Log optimization methodology, and clearly show that this methodology succeeds where the traditional mean-variance methodology fails to deliver the kinds of optimal portfolios that investors desire.

Figure 3: Mean-Variance Efficient Portfolios



5. Summary and Conclusions

The Power-Log optimization methodology works for all portfolio construction applications without limitations. In this study we have simulated the joint return distributions used for optimization. Depending on the application, it is possible to use empirical distributions when there is sufficient historical data available. For example here we have assumed annual rebalancing, which allows us to use a lognormal distribution for modeling S&P500 index returns. A fund manager, who trades daily, would have to use a daily return distribution, which has far more kurtosis than the annual return distribution. In this case a lognormal approximation would be inappropriate, but the manager could use historical daily returns to construct an empirical joint return distribution. Similarly, for asset allocation using market indexes, or funds, a joint return distribution based on historical returns could be appropriate as well.

In our examples we have used a treasury security, the S&P500 index and the closest to the money call option on the index. The asset list can be expanded easily, to include in the money and out of the money call and put options on the S&P500 index, other indexes and assets, and put and call options on them. In addition, managers who are constrained to holding long positions in the underlying assets can use the method to construct optimal overlay strategies with options. This flexibility allows Power-Log optimization to be used for all types of portfolio construction applications.

For portfolios containing assets such as stocks, bonds and options, whose return distributions are normal and nonnormal, Power-Log optimization offers a unique approach to constructing portfolios that conform closely to investor preferences by taking advantage of the skewness and kurtosis in returns. Power-Log utility functions combine tenets of behavioral finance with multiperiod portfolio theory to represent investor preferences realistically, and model the entire range of investor preferences from high to low tolerance for risk. Using Power-Log utility functions to optimize portfolios containing a treasury security, the S&P500 index and a call option on the index, we show that for virtually the entire range of investor preferences, optimal

Power-Log portfolios deliver higher geometric average realized returns with lower downside risk than mean-variance efficient portfolios that have the same ex-ante expected returns. Optimal Power-Log portfolios also provide much better downside protection against large unanticipated market downturns, as in 2002 and 2008 than corresponding mean-variance efficient portfolios. Our findings clearly show that Power-Log optimization succeeds in managing downside risk effectively in contrast to traditional mean-variance optimization, which fails to deliver the kinds of optimal portfolios that investors desire.

References

Anagnostopoulos, KP and Mamanis, G 2010, 'Using Multiobjective Algorithms to Solve the Discrete Mean-Variance Portfolio Selection,' *International Journal of Economics and Finance*, Vol. 2, No. 3, pp. 152-162.

Barberis, N and Thaler R 2003, 'A Survey of Behavioral Finance,' *Handbook of the Economics of Finance*, pp. 1053-1128, Elsevier.

Bawa, VS 1978, 'Safety-first, Stochastic Dominance, and Optimal Portfolio Choice,' *Journal of Financial and Quantitative Analysis*, Vol. 13, No. 2, pp. 255-271.

Best, MJ, and Ritter, K 1976, 'A Class of Accelerated Conjugate Direction Methods for Linearly Constrained Minimization Problems,' *Mathematics of Computation*, Vol. 30, No. 135, pp. 478-504.

Cremers, J, Kritzman, M and Page, S 2005, 'Optimal hedge Fund Allocations: Do Higher Moments Matter?' *The Journal of Portfolio Management*, Vol. 31, No. 3, pp. 70-81.

Fishburn, PC 1977, 'Mean-Risk Analysis with Risk Associated with Below-Target Returns,' *The American Economic Review*, Vol. 67, No. 2, pp. 116-126.

Friedman, M and Savage, LJ 1948, 'The Utility Analysis of Choices Involving Risk,' *The Journal of Political Economy*, Vol. 56, No. 4, pp. 279-304.

Grauer, RR and Hakansson, NH 1982, 'Higher Return, Lower Risk: Historical Returns on Long-Run, Actively Managed Portfolios of Stocks, Bonds and Bills,' 1936-1978, *Financial Analysts Journal*, Vol. 38, pp. 39-53.

Kahneman, D and Tversky, A 1979, 'Prospect Theory: An Analysis of Decision Under Risk,' *Econometrica*, Vol. 47, pp. 263-291.

Jarrow, R and Zhao, F 2006. 'Downside Loss Aversion and Portfolio Management,' *Management Science*, Vol. 52, No. 4, pp. 558-566.

Kale, JK 2006, 'Growth Optimization with Downside Protection: A New Paradigm for Portfolio Selection,' *The Journal of Behavioral Finance*, Vol. 7, No. 1, pp. 146-150.

Kale, JK 2011, '*A Deterministic Simulation Methodology for Evaluating Optimal Power-Log Utility and Other Portfolios when Asset Returns are Skewed*,' Presented at the Fortieth Annual Meeting of the Western Decision Sciences Institute.

Kale, JK and Sheth, A 2012, '*Power-Log Optimization for Maximizing Portfolio Growth and Controlling Tail Risk*,' Working Paper, Saint Mary's College of California.

Kelly, JL 1956, 'A New Interpretation of Information Rate, *Bell Systems Technical Journal*,' Vol. 35, pp. 917-926.

Leland, H 1999, 'Beyond Mean-Variance: Performance Measurement in a Nonsymmetrical World,' *Financial Analysts Journal*, Vol. 55, No. 1, pp. 27-36.

Leland, H and Rubinstein, M 1988, '*The Evolution of Portfolio Insurance. Dynamic Hedging: A Guide to Portfolio Insurance*,' ed. Don Luskin, John Wiley and Sons.

Markowitz, HM 1952, 'The Utility of Wealth,' *Journal of Political Economy*, Vol. 60, pp. 151-158.

Markowitz, HM 1952, 'Portfolio Selection,' *Journal of Finance*, Vol. 8, pp. 77-91.

Markowitz, HM 1959, 1991, '*Portfolio Selection*,' Blackwell.

Markowitz, HM 1976, 'Investment for the Long Run: New Evidence for an Old Rule,' *Journal of Finance*, Vol. 31, pp. 1273-1286.

Pedersen, C 2001, 'Derivatives and Downside Risk,' *Derivatives Use, Trading and Regulation*, Vol. 7, No. 3, pp. 251-268.

'RiskMetrics – Technical Document,' J. P. Morgan, December 17, 1996.

Rockafellar, RT and Uryasev, S 2000, 'Optimization of Conditional Value-at-Risk,' *Journal of Risk*, Vol. 2, pp. 21-41.

Rubinstein, M 1991, 'Continuously Rebalanced Investment Strategies,' *Journal of Portfolio Management*, Vol. 18, No. 1, pp. 78-81.

Rubinstein, M and Leland, H 1981, 'Replicating Options with Positions in Stock and Cash,' *Financial Analysts Journal*, Vol. 37, No. 4, pp. 63-72.

Samuelson, PA 1971, 'The Fallacy Of Maximizing The Geometric Mean in Long Sequences of Investing or Gambling,' *Proceeding of the National Academy of Science USA*, Vol. 68, pp. 2493-2496

Savage, L 1964, '*The Foundations of Statistics*,' Wiley, New York.

Tversky, A and Kahneman, D 1991, 'Loss Aversion in Riskless Choice: A Reference-Dependent Model,' *Quarterly Journal of Economics*, Vol. 106, pp. 1039-1061.

Vercher, E, Bermudez JD and Segura JV 2007, 'Fuzzy Portfolio Optimization Under Downside Risk Measures,' *Fuzzy Sets and Systems*, pp. 769-782, Elsevier.

Von Neumann, J and Morgenstern, O 1944, '*Theory of Games and Economic Behavior*,' Princeton University Press, Princeton, New Jersey.